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INFLUENCE OF THE PLASTICITY OF A JOINING MATERIAL ON THE KINK OF AN INTERFACE CRACK AT THE CORNER POINT OF THE INTERFACE OF MEDIA

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We present the solution of the problem of process zone which develops under the condition of plain deformation from the tip of an interface crack that coincides with the corner point of the interface of two different materials. The process zone is modeled by the line of discontinuity of tangential displacements on the interface and by the lateral line of discontinuity of the normal displacements. The equation for finding the lengths of both discontinuity lines and the expression for the crack opening displacement are obtained. We study the influence of plasticity of the material of the joint on the direction and conditions of crack start.

Keywords: corner point of the interface of media, interface crack, interface plastic strip, lateral process zone, crack kinking.

Plasticity plays an important role in the strength of materials promoting their unloading in the vicinity of stress concentrators and, as a result, the increase in the limiting equilibrium loads [1]. In [2], the kink of an interface crack propagating from the corner point of the interface of two different media was studied by neglecting the plasticity of the material. In what follows, we study the influence of the plasticity of a joining material on the initial kink of the crack regarded as a result of two subsequent processes. It is assumed that a narrow plastic strip propagates in the initial stage from the crack tip along the interface and changes the stress-strain state in the vicinity of the crack but does not remove stress concentration. This is why, in the next stage, in the less crack-resistant material of the composite, we observe the formation of a lateral process zone inclined to the interface, and, after the attainment of the critical opening displacement at the initial point (crack tip), the interparticle bonds break and the crack starts. This means that it is necessary to successively solve the following two problems: In the first problem, according to the Leonov–Panasyuk model, we determine the sizes of the initial plastic strip on the interface of the materials and the stress-strain state in the vicinity of the crack tip after the formation of the strip. These characteristics are used for the solution of the problem of secondary lateral process zone and to establish the conditions of crack start.

Parameters of the Initial Plastic Strip in the Joining Material

Under the conditions of plane deformation, we consider the problem of a small plastic strip in the vicinity of the tip of an interface crack propagating from the corner point of the interface of two homogeneous isotropic media with Young's moduli E_1 and E_2 and Poisson's ratios ν_1 and ν_2, respectively. Assume that the

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joining material is more plastic than the materials of the joined parts of the body. Thus, neglecting the thickness of the joining material, we model the plastic strip by the line of discontinuity of tangential displacements in which the tangential stress is equal to the shear yield strength of the joining material τ_s [3].

Assuming that the length of the initial plastic strip l is much smaller than the crack length L and all other sizes of the body, for the determination of the parameters of the strip and the stress-strain state in the vicinity of the strip, we consider the body as a piecewise homogeneous plane with interface of the media in the form of the sides of an angle from the vertex of which a semiinfinite crack whose lips free of loads propagates along one of the sides of the angle and the line of discontinuity of tangential displacements of finite length propagates along the other side. At infinity, we formulate the condition of coincidence of the desired solution with the solution of a similar problem of the theory of elasticity when the line of discontinuity is absent [4], which is characterized by the roots λ of the equation

$$D(\lambda) = 0\,, \tag{1}$$

$$D(\lambda) = -(1+\kappa_1)^2 d_1 - 4(1+\kappa_1)(e-1)d_1 d_2 - e^2(1+\kappa_2)^2 d_3$$

$$+\,4(e-1)^2 d_1 d_3 + 4e(1+\kappa_2)(e-1)d_3 d_4 + 2e(1+\kappa_2)(1+\kappa_1)d_5\,,$$

$$d_1 = (\lambda+1)^2 \sin^2\alpha - \sin^2(\lambda+1)\alpha\,,$$

$$d_2 = \sin^2(\lambda+1)(2\pi-\alpha)\,, \qquad d_3 = (\lambda+1)^2\sin^2\alpha - \sin^2(\lambda+1)(2\pi-\alpha)\,,$$

$$d_4 = \sin^2(\lambda+1)\alpha\,, \qquad d_5 = d_1 + \sin(\lambda+1)\alpha\sin 2\lambda\pi\cos(\lambda+1)(2\pi-\alpha)\,;$$

$$e = \frac{1+\nu_2}{1+\nu_1}\frac{E_1}{E_2}\,, \qquad \kappa_{1(2)} = 3-4\nu_{1(2)}\,.$$

By using the Wiener–Hopf method [5], we find the solution of the corresponding boundary-value problem just as the solution of a similar problem of the interface process zone, which was modeled by the line of discontinuity of the normal displacement [6]. As a result, we obtained the following transcendental equations for the determination of the relative length $x = l/L$ of the plastic bend depending on the roots of the characteristic equation (1) satisfying the condition $-1 < \operatorname{Re}\lambda < 0$:

(a) characteristic equation with two or three real roots:

$$\sum_i n_i F(\lambda_i) N(\lambda_i) x^{\lambda_i} = \operatorname{sgn}(F(\lambda_1))\frac{N(0)}{\tau}\,, \tag{2}$$

$$n_i = \frac{C_i}{C_1}\frac{L^{\lambda_i}}{L^{\lambda_1}}\,, \qquad N(\lambda) = \frac{K^+(-\lambda-1)}{(\lambda+1)G^+(-\lambda-1)}\,, \qquad \tau = \frac{|C_1|\,L^{\lambda_1}}{\tau_s}\,;$$