Theorem 4. *If $p_{ii} > 0$, $p_{i(i+1)} > 0$ and $p_{ii} + p_{i(i+1)} = 1$ $\forall i \in \mathbb{N}$, then the spectrum of the distribution of the r.v. θ is an anomalously fractal set.*

In the report we offer results of studying of properties of the distribution of r.v. θ such that $\theta_1, \theta_3, \theta_5, \ldots$ are independent and $\theta_2, \theta_4, \theta_6, \ldots$ are elements of Markov chain.

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INVERSOR OF DIGITS OF Q_3-REPRESENTATION OF FRACTIONAL PART OF REAL NUMBER AS A DISTRIBUTION FUNCTION OF RANDOM VARIABLE

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Let $Q_3 = \{q_0, q_1, q_2\}$ be a fixed set of positive real numbers such that $q_0 + q_1 + q_2 = 1$. It is known [1] that for any number $x \in [0, 1)$ there exists sequence (α_n), $\alpha_n \in A_3 = \{0, 1, 2\}$, such that

$$x = \beta_{\alpha_1} + \sum_{k=2}^{\infty} \left[\beta_{\alpha_k} \prod_{j=1}^{k-1} q_{\alpha_j} \right] = \Delta^{Q_3}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}, \qquad (1)$$

where $\beta_0 = 0$, $\beta_1 = q_0$, $\beta_2 = q_0 + q_1$.

Brief notation $\Delta^{Q_3}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}$ of series (1) is called Q_3-representation of number x. This is classic ternary representation if $q_0 = q_1 = q_2$.

If we do not use representation with period (2), then any number $x \in [0, 1)$ has a unique Q_3-representation. Thus function $I(\Delta^{Q_3}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}) = \Delta^{Q_3}_{[2-\alpha_1][2-\alpha_2]\ldots[2-\alpha_n]\ldots}$ is well defined. It is called *inversor of digits of Q_3-representation of number $x \in [0, 1)$.*

Theorem 1. *For given Q_3-representation of number, function $F(x) \equiv 1 - I(x)$ is a distribution function of random variable $\xi = \Delta^{Q_3}_{\eta_1\eta_2\ldots\eta_k\ldots}$, where η_k are independent random variables taking the values $0, 1, 2$ with probabilities q_2, q_1, q_0 respectively. $F(x) = x$ if $q_0 = q_2$, and $F(x)$ is singular strictly increasing probability distribution function of Salem type if $q_0 \neq q_2$.*

In the talk, we also study other properties of inversor (in particular, fractal properties) and propose finite system of functional equations such that this function is a unique solution of this system in the class of continuous functions.

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ON PROBABILISTIC APPROACH TO GDP TRANSFORMATIONS

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It is well known that methods of fractal geometry are extremely useful in the theory of singularly continuous probability measures, because such measures are usually supported by fractals (see, e.g., [2] and references therein). On the other hand probabilistic methods are among the most powerful tools to study fractals. During the last decade the theory of transformations preserving the Hausdorff-Besicovitch dimension (DP-transformations) is shown to be very useful for the calculation of Hausdorff-Besicovitch dimension of fractals with a non-regular local structure (see, e.g., [1]). Probabilistic approach to DP-transformations was developed in papers by S. Albeverio, M. Ibrahim, M. Lebid, M. Pratsiovytyi, G. Torbin and others.

In particular, it has been shown that the family of all continuous DP-transformations of the unit interval is essentially wider than the family of bi-Lipschitz transformations. Such transformations are also important from the group theoretical point of view, because they allow us to consider fractal geometry as a special field of mathematics studying invariants of the DP-group. On the other hand, Yu.Peres and G. Torbin have shown that even strictly increasing continuous DP-transformations are unstable under convex combinations.

In the talk we shall introduce and discuss properties of generalized DP-transformations. We shall paid a special attention to a relatively stable subgroup of GDP-transformations consisting of transformations preserving triviality and non-triviality of the Hausdorff measure.

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NEGA-$\tilde{Q}$-REPRESENTATION OF REAL NUMBERS

S. O. Serbenyuk

Let $\tilde{Q} = ||q_{i,j}||$ be a fixed matrix, where $i = \overline{0, m_j}$, $m_j \in N^0_\infty = \mathbb{N} \cup \{0, \infty\}$, $j = 1, 2, ...$, such that:
1. $\mathbb{R} \ni q_{i,j} > 0$;
2. $\forall j \in \mathbb{N} : \sum_i q_{i,j} = 1$;
3. $\forall (i_j), i_j \in \mathbb{N} \cup \{0\}$: $\prod_{j=1}^{\infty} q_{i_j,j} = 0$.

Definition 1. A representation of $x \in [0; 1]$ by the expansion

$$x = a_{i_1(x),1} + \sum_{k=2}^{\infty} \left[a_{i_k(x),k} \prod_{j=1}^{k-1} q_{i_j(x),j} \right],$$

where $a_{i_k,k} = \sum_{i=0}^{i_k - 1} q_{i,k} \neq 0$ for $i_k \neq 0$ and $a_{0,k} = 0$, is the $\tilde{Q}$-*expansio*n of $x \in [0; 1]$, and it is denoted by $x = \Delta^{\tilde{Q}}_{i_1(x) i_2(x) \ldots i_k(x) \ldots}$ [1]. This notation is $\tilde{Q}$-*representation* of x.

Definition 2. The representation $\Delta^{-\tilde{Q}}_{i_1 i_2 \ldots i_k \ldots}$ such that

$$x = \Delta^{-\tilde{Q}}_{i_1(x) i_2(x) \ldots i_k(x) \ldots} \equiv \Delta^{\tilde{Q}}_{i_1(x)[m_2 - i_2(x)] i_3(x)[m_4 - i_4(x)] \ldots} \equiv a_{i_1(x),1} + \sum_{k=2}^{\infty} \left[\tilde{a}_{i_k(x),k} \prod_{j=1}^{k-1} \tilde{q}_{i_j(x),j} \right],$$

where $\tilde{a}_{i_k(x),k} = a_{m_k - i_k(x),k}$, $\tilde{p}_{i_k,k} = p_{m_k - i_k(x),k}$ for an even number k and $\tilde{a}_{i_k(x),k} = a_{i_k(x),k}$, $\tilde{p}_{i_k(x),k} = p_{i_k(x),k}$ for an odd number k, is called *nega-$\tilde{Q}$-representation* of a number $x \in [0; 1]$.

The talk is devoted to formulation of a foundation of metric theory of real numbers nega-$\tilde{Q}$-representation.

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ON SINGULAR DISTRIBUTION OF RANDOM VARIABLES WITH INDEPENDENT SYMBOLS OF F-EXPANSION

Lilia Sinelnyk

Let $x = \Delta^F_{i_1(x) i_2(x) \ldots i_n(x) \ldots}$ be a generalized F-expansion of real numbers over an alphabet A. In such a general case we fix only two simple assumptions on the family of corresponding cylinders:
1) interior parts of cylinders of the same rank do not intersect;
2) $\mid \triangle_{i_1 i_2 \ldots i_n} \mid \to 0$ $(n \to \infty)$.

Let $\xi_1(x), \xi_2(x), ..., \xi_n(x)...$ be a sequence of independent random variables and $\xi = \Delta^F_{\xi_1(x) \xi_2(x) \ldots \xi_n(x) \ldots}$ be a random variable with independent symbols of F-expansion, $P\{\xi_k = i_0\} = p_{i_0 k}$.

Theorem 1. *If there exist a symbol $i_0 \in A$, such that:*
1) $\sum\limits_{k=1}^{\infty} p_{i_0 k} = +\infty$,
2) for any number x from interval $[0, 1]$ and for any $n \in N$ there exists a sequence $c_n = c_n(i_0)$:

$$\frac{\left| \Delta^F_{i_1(x) i_2(x) \ldots i_{n-1}(x) i_0} \right|}{\left| \Delta^F_{i_1(x) i_2(x) \ldots i_{n-1}} \right|} \leq c_n(i_0) \quad and \quad \sum_{n=1}^{\infty} c_n < +\infty,$$

then
1) λ-almost all $x \in [0, 1]$ contain symbol i_0 only finitely many times;
2) the probability measure μ_ξ is singular with respect to Lebesgue measure.