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# THE USE OF THE "TRIDENT" MODEL IN THE ANALYSIS OF PLASTIC ZONES NEAR CRACK TIPS AND CORNER POINTS

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**The paper deals with calculation of a plastic zone near a crack tip in a homogeneous elastoplastic solid and near a corner point of the boundary of this solid. The calculations are conducted for a solid subject to plane strain and within the framework of models with plastic strips. It is shown that in comparison with the widely used model with two straight slip-lines, the process of plastic deformation is described by the "trident" model more accurately. The results of calculations of the plastic zone by the "trident" model that correspond to different stages of the development of plastic deformation are given for a crack of normal separation in a quasibrittle material.**

Problems arising in studying the fracture of elastoplastic [2–6, 10, 11], viscoelastic [12], and inhomogeneous [13, 15] materials call for the development of more perfect models of cracks representing the actual pattern of the fracture process.

A great number of papers devoted to the calculation of plastic zones near crack tips under plane deformation have been published in recent years. In these studies, the plastic zones are modeled by two narrow rectilinear plastic strips coming from the crack tip and representing slip lines [6, 8, 10]. On a slip line, only the tangential displacement may have a discontinuity, while the tangential stress is equal to the shear yield point $\tau_s$.

The basis for such a simulation is the results of the experimental investigation [5]. According to this study, at the initial stage of plastic deformation, two plastic strips appear near a tip of a crack of normal separation. The strips are tilted to the line of crack continuation at an angle of approximately 72° [10]. In [10], it is shown that the results of calculation of the initial plastic zone near a tip of a crack of normal separation within the framework of the mentioned model comply, in some sense, with those of numerical calculation of a "fuzzy" plastic zone having the shape of plastic "ears" [14]. Therefore, the plastic zone corresponding to this model can be considered as some approximation of a "fuzzy" plastic zone.

However, as follows from the investigations of the plastic zone near the crack tips using an electron microscope and the x-ray diffraction method [2, 3], at all stages of the fracture process, there exists a frontal prefracture zone including the initial one along with strongly developed "butterfly"-shaped side plastic zones. The linear dimension of this zone at the continuation of the crack is much less than the maximum linear dimensions of the side elastic zones. With an increase in the load, the plastic prefracture zone turns to a destruction zone distinguished by the maximum level of plastic strains and the presence of pores and microcracks. As the load increases, the plastic zones grow and the angle of inclination of side plastic zones changes and achieves 50° at subsequent stages of the fracture process.

The existence of the third plastic zone (along with the two side ones) that develops from a crack tip can be proved not only in an experimental but also in a theoretical way. To this end, it is sufficient to examine the behavior of the stresses near a crack tip in the symmetric problem [10] of the theory of elasticity for a plane whose point is the origin of a semiinfinite crack and two finite slip lines. This study was carried out in [1]. Its results show that after the occurrence of side plastic slip lines coming from the tip of a crack of normal separation, the crack tip remains a concentrator of stresses with a power singularity, though weaker than the crack tip in an elastic body. The degree of singularity of the stresses depends on the angle of inclination of the slip line to the line of the crack continuation. If the angle is equal to 72°, then the degree of singularity

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is approximately equal to 0.20049. The presence of the mentioned stress concentration implies that once two side plastic zones coming from a crack tip appear, the third plastic zone may be expected to develop from it.

The symmetric problem of the theory of elasticity is solved in [1] for a plane whose point is the origin of a semiinfinite crack, two slip lines of finite length, and the Dugdale line of considerably smaller length, on which only normal shear rupture is possible, and the direct stress is equal to the tension yield point $\sigma_s$. The examination of the stress behavior near a crack tip in the given problem shows that once the third plastic line coming from a tip of a normal-separation crack appears, the crack tip is not a stress concentrator any more. Therefore, except for the three plastic zones developing from the crack tip, new plastic zones may not be expected.

These experimental and theoretical results suggest that, as compared to a model with two slip lines, the "trident" model more accurately describes the process of plastic deformation near a crack tip. According to this model, the set of three narrow rectilinear plastic strips (two slip lines and the Dugdale line) coming from a crack tip models a plastic zone near the crack tip. If, at the initial stage, the plastic strains near the crack tip are localized in thin layers of the material — three plastic strips coming from the tip — then, at the subsequent stages of plastic-strain development, the plastic zone corresponding to the "trident" model should be considered as some approximation of the actual "fuzzy" plastic zone observed in the above-mentioned experiments [2, 3] for a crack of normal separation.

For hardenable materials, it is desirable to introduce the parameter $\sigma_B$ (the ultimate strength of a material) instead of $\sigma_s$, since, as follows from [2, 3], the strains in the prefracture zone achieve an extremely high level (up to 50%) at the stages of continuation and growth of the crack, and, hence, the stresses in this zone considerably exceed the yield point of the material.

The initial plastic zone near the tip of a crack of normal separation is calculated in [1] within the framework of the "trident" model. However, for a detailed study of the development of plastic strains near the tip of a normal-separation crack, it is necessary to have the values of the length $l$ of the slip lines, the length $d$ of the Dugdale line, and the opening $\delta$ of the crack at its tip, which correspond to subsequent stages of this process.

The mentioned quantities $l$, $d$, and $\delta$ are evaluated below for different values of the angle $\alpha$ between the slip line and the line of continuation of a crack of normal separation in a quasibrittle material.

According to [1], we obtain the following formulas for determination of the lengths of plastic strips:

$$l = L(\alpha)\frac{K_I^2}{\tau_s^2}, \quad L = \frac{\pi}{32}\left[\frac{G_1^+(-1)}{G_1^+(-1/2)}\right]^2 \sin^2\alpha\cos^2\frac{\alpha}{2},$$

$$d = D^*\frac{K_I^2}{\tau_s^2}, \quad D^* = D(\alpha)\left(\frac{\tau_s}{\sigma_s}\right)^{-1/\lambda}, \quad D = \frac{\pi}{32}D_0,$$

$$D_0 = \left(\frac{S^*}{2\lambda+1}\right)^{-1/\lambda}\left[\frac{G_1^+(-1)}{G_1^+(-1/2)}\right]^2\left[\frac{G_1^+(-\lambda-1)G_2^+(-1)}{G_1^+(-1)G_2^+(-\lambda-1)}\right]^{-1/\lambda}\sin^2\alpha\,\cos^2\frac{\alpha}{2},$$

$$s^* = \frac{4(\lambda+2)\sin(\lambda+1)(\pi-\alpha)\sin\lambda\pi\sin\alpha}{\lambda s},$$

$$s = -2\pi\sin 2\lambda\pi + [2(\pi-\alpha)\cos 2\alpha - \sin 2\alpha]$$

$$\times\sin 2(\lambda+1)(\pi-\alpha) + 4(\alpha\cos\alpha+\sin\alpha)(\lambda+1)$$

$$\times\cos 2(\lambda+1)\alpha\sin\alpha - [(2\alpha+\sin 2\alpha)\lambda(\lambda+2)$$

$$-(2\alpha\cos 2\alpha+\sin 2\alpha)(\lambda+1)^2]\sin 2(\lambda+1)\alpha$$

$$-4(\pi-\alpha)(\lambda+1)\cos 2(\lambda+1)(\pi-\alpha)\sin\alpha\cos\alpha + 4(\lambda+1)\sin^2\alpha,$$