International Applied Mechanics, Vol. 31, No. 6, 1995

SLIP LINES AT THE END OF A CUT AT THE INTERFACE OF DIFFERENT MEDIA

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UDC 539.375

In the conditions of a plane static problem, the initial plastic zone close to a cut with stress-free sides at the interface of different media is considered. The plastic zone is modeled by a slip line starting from the end of the cut and lying at the boundary of the media. This situation arises when the materials of the two contacting bodies are considerably more rigid than the binder (glue) material at the boundary of the two media where the crack appears. The interaction of the sides of the slip line is modeled on the basis of dry Coulomb friction with adhesion [7]. Taking account of the shortness of the slip line, the corresponding elasticity-theory boundary problem is formulated for a piecewise-homogeneous plane with a semiinfinite rectilinear cut and a straight slip line at its end. At infinity, a condition allowing the influence of the external field on the stress–strain state of a region with an infinitely remote point to be taken into account is imposed. This condition includes the stress-intensity coefficients at the end of the cut. An integral Mellin transformation is used to reduce the problem to a Wiener–Hopf functional equation. The accurate solution of the functional equation is constructed in terms of Cauchy-type integrals and gamma functions. The stress-intensity coefficient at the end of the slip line is determined and, on this basis, an equation for determining the length of the slip line is derived. It establishes a relation between the length of the slip line and the stress-intensity coefficient at the end of the line.

1. FORMULATION OF THE PROBLEM

Consider the plane static problem of the initial plastic zone close to the end of a cut at the interface of two media, with Young's moduli E_1, E_2 and Poisson's ratios ν_1, ν_2. The plastic zone is modeled by a slip line starting from the end of the cut and also lying at the interface of the media. The sides of the cut are stress-free, and the interaction of the sides of the slip line is described on the basis of dry Coulomb friction with adhesion. Since the length of the slip line is small in comparison with the length L of the cut and all the other dimensions of the region, and the stress–strain state is only investigated close to the end of the cut, the solution of the given problem will be taken in the form of the solution of the corresponding elasticity-theory problem for a plane consisting of two different homogeneous halfplanes containing a semiinfinite cut at the interface of the media, with a slip line starting from the end of the cut (Fig. 1). At infinity, the principal terms in the series expansion of the stress correspond to the asymptotically greatest solution of the analogous problem with no slip line; this solution satisfies the stress-damping condition, and is given in [2], for example. It is oscillatory close to the end of the cut and contains arbitrary constants: the stress-intensity coefficients at the end of the cut established from the solution of the external problem, i.e., the problem for the initial finite region with no slip line. In the problem considered here, with a slip line at the end of the semiinfinite cut (the internal problem), however, these stress-intensity coefficients are assumed to be specified according to a certain condition. They characterize the intensity of the external field. The problem as a whole (including both the external and internal problems) is a problem for an initial finite region with a slip line.

In this formulation (an asymptotic problem regarding the fine structure at the end of the crack), the problem of slip lines at the end of a cut in a homogeneous region was solved in [8].

Close to the end of the slip line, there is an asymptote corresponding to the asymptotically greatest solution of the problem for a plane consisting of two different homogeneous halfplanes with a semiinfinite slip line at the interface of the media, under the corresponding homogeneous boundary conditions; this solution satisfies the condition of continuity of the displacements. The method of solving such homogeneous problems of elasticity theory was described in [5], for example. It

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, Kiev. Uman Pedagogical Institute, Ukraine. Translated from Prikladnaya Mekhanika, Vol. 31, No. 6, pp. 86-91, June, 1995. Original article submitted March 10, 1994.

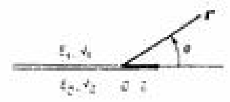

Fig. 1

yields a transcendental characteristic equation, each of the many roots λ of which corresponds to a solution of the form $f(\theta)r^{\lambda}$ for the stress. The solutions determined include arbitrary constants. The solution given here corresponds to a single root of the characteristic equation in the interval $]-1; 0[$: $\lambda_0 = 1/\pi \arccos\left[e_2/\sqrt{e_1^2 - e_2^2}\right] - 1$, $e_1 = \chi_1 + e + e\chi_2 + 1$, $e_2 = k(\chi_1 + e - e\chi_2 - 1)$, $\chi_{1,2} = 3 - 4\nu_{1,2}$, $e = E_1(1 + \nu_2)/[E_2(1 + \nu_1)]$ (k is the frictional coefficient). This root includes one arbitrary constant with the dimensions of force divided by length to the power $(\lambda_0 + 2)$, the stress-intensity coefficient at the end of the slip line, which is found later in the process of solving the problem. Thus, in the present case, the degree of singularity of the stress close to the end of the slip line differs from the traditional value $-1/2$ and is λ_0.

The boundary conditions of the problem are as follows

$$\theta = \pm\pi,\ \sigma_\theta = \tau_{r\theta} = 0;\ \theta = 0,\ \langle\sigma_\theta\rangle = \langle\tau_{r\theta}\rangle = 0,\ \langle u_\theta\rangle = 0; \tag{1.1}$$

$$\theta = 0,\ r < l,\ \tau_{r\theta} = k_1 - k\sigma_\theta;\ \theta = 0,\ r > l,\ \langle u_r\rangle = 0; \tag{1.2}$$

$$\theta = 0,\ r \to l - 0,\ \frac{E_1}{4(1 - \nu_1^2)}\left\langle\frac{\partial u_r}{\partial r}\right\rangle \sim -K\frac{(\chi_1 + e)(1 + e\chi_2)}{(1 + \chi_1)^2}\left[2\pi(l - r)\right]^{\lambda_0};$$

$$\theta = 0,\ r \to l + 0,\ \tau_{r\theta} + k\sigma_\theta \sim K\frac{\sqrt{e_1^2 + e_2^2}}{2(1 + \chi_1)}\left[2\pi(l - r)\right]^{\lambda_0}; \tag{1.3}$$

$$\theta = 0,\ r \to \infty,\ \tau_{r\theta} + k\sigma_\theta = F(r) + \overline{F(r)} + O\left(\frac{1}{r}\right) \tag{1.4}$$

$$F(r) = e'[K_{II} + kK_I + i(kK_{II} - K_I)]L^{i\omega}r^{-1/2 + i\omega};$$

$$e' = \frac{e_1}{4\sqrt{2\pi(\chi_1 + e)(1 + e\chi_2)}},\ w = \frac{1}{2\pi}\ln\frac{\chi_1 + e}{1 + e\chi_2}.$$

Here σ_θ, $\tau_{r\theta}$ denote the stress; u_θ, u_r denote the displacement; $\langle a\rangle$ denotes a discontinuity in a; k_1 is the adhesion coefficient; K_1, K_{11} are specified stress-intensity coefficients at the end of the cut; K is the stress-intensity coefficient at the end of the slip line, which remains to be determined; $\bar{W}$ is the complex conjugate of W.

The solution of the problem is expressed as the sum of solutions of the following two problems. The first (problem A) differs from that above in that the first condition in Eq. (1.2) is replaced by

$$\theta = 0,\ r < l,\ \tau_{r\theta} + k\sigma_\theta = k_1 - F(r) - \overline{F(r)}, \tag{1.5}$$

and the stress declines at infinity as $o(1/r)$; in particular, the first two terms on the right-hand side of Eq. (1.4) are missing. The second problem (problem B) is the analogous problem without the slip line. Since the solution of problem B is known, it is sufficient to construct the solution of problem A.

2. SOLUTION OF WIENER–HOPF EQUATION

Applying the Mellin transformation

$$m^*(p) = \int_0^\infty m(r)r^p\,dr$$