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MODEL OF THE FRACTURE PROCESS ZONE AT THE TIP OF A CRACK REACHING THE NONSMOOTH INTERFACE BETWEEN ELASTIC MEDIA

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The paper is concerned with the fracture process zone at the tip of a crack at the nonsmooth interface between isotropic elastic media. A plane symmetric problem is formulated. The zone is modeled by lines of discontinuity of the normal displacement at the interface. The exact solution of the elastic problem is found by the Wiener–Hopf method

Keywords: fracture process zone, interface, crack, Wiener–Hopf method

Introduction. The high crack resistance of composites is due to crack growth inhibition at the interfaces between the components of these materials [15, 16]. This is why studies of cracks growing to reach the interface of dissimilar media are of current importance. While problem solutions describing the growth of cracks located at interfaces of various media are addressed in the extensive literature [1, 2, 4, 5, 8, 13, 17, 20, 21], studies of cracks growing to reach such interfaces are much fewer.

A crack reaching the interface between media with different mechanical properties may cause partial separation of the materials (fracture process zone) because of the high stress intensity at its tip, which was demonstrated in many studies. Such fracture process zones are usually modeled by discontinuity lines of the normal displacement in the case of brittle matrix (generalized Leonov–Panasyuk model [1, 8, 11]) or plastic slip lines in the case of plastic matrix (Cherepanov's modified model [3, 5, 17]). To gain insight into the failure mechanism of piecewise-homogeneous media, it is important to know the stress intensity at the crack tip, the type of its singularity, the size of the fracture process zones, and the way it affects the stress intensity. Knowing the singularities of stresses at the tip of a crack reaching an interface, we can more efficiently develop numerical methods for determining the stress state of compound piecewise-homogeneous bodies with cracks. Solving such problems is also needed for the development of the theory of failure of adhesive joints with allowance for the rheological properties of adhesives (plastic, brittle, viscoelastic, etc.)

Symmetric plane solutions describing the fracture process zone at the tip of a crack reaching the nonsmooth interface of media that comprise a piecewise-homogeneous elastoplastic body were found in [3, 18] using plastic slip lines as models. We will solve a similar problem considering that the fracture process zone (partial separation zone) is modeled by discontinuity lines of the normal displacement at the interface (brittle matrix).

1. Problem Formulation. Consider a piecewise-homogeneous isotropic elastic body under plane strain. It consists of different homogeneous parts with a thin binder interlayer between them. The interlayer is more brittle than the bonded parts. Let the tip of a rectilinear crack be at a corner point of the nonsmooth interface between two parts of the body. This domain is considered symmetric about the crack line.

As the external load increases, a fracture process zone appears and grows at the crack tip. We will study only the initial stage of its development, considering that the external load is rather low. Then the zone is much smaller than the crack and the body.

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Since the fracture process zone is small, we have a plane static symmetric problem of elasticity for a piecewise-homogeneous isotropic plane with the interface in the form of the sides of an angle containing a semi-infinite crack emanating from the vertex and having a fracture process zone. At infinity, we have an asymptotics which is the solution of a similar problem without a fracture process zone generated by the minimum (within]–1; 0[) root of its characteristic equation. This problem (problem K) was considered in [10]. The arbitrary constant appearing in the solution is assumed given. It characterizes the intensity of the external field and should be determined from the solution of the external problem.

It is assumed that in the elastic problem for a finite body at this stage of deformation (no fracture process zone has appeared yet) (problem I), the normal stress at the crack tip, which is at the interface, is tensile (condition T). The constraints for the problem parameters that provide this condition will be established below.

Since the binder is assumed elastobrittle, the strains in the fracture process zone follow the separation mechanism. Therefore, a strip-like zone is modeled by a discontinuity line of the normal displacement on which the normal self-balanced stresses are equal to a preset constant σ (separation resistance) of the binder [11].

Thus, the boundary conditions of the elastic problem modeling the process involved can be expressed as follows considering symmetry (Fig. 1):

$$\theta=\pi-\alpha,\quad \sigma_\theta=\tau_{r\theta}=0,\qquad \theta=-\alpha,\quad \tau_{r\theta}=0,\quad u_\theta=0,$$

$$\theta=0,\quad \langle\sigma_\theta\rangle=\langle\tau_{r\theta}\rangle=0,\qquad \langle u_r\rangle=0, \tag{1.1}$$

$$\theta=0,\quad r<1,\quad \sigma_\theta=\sigma;\quad \theta=0,\quad r>1,\quad \langle u_\theta\rangle=0, \tag{1.2}$$

$$\theta=0,\quad r\to\infty,\quad \sigma_\theta=Cgr^\lambda+o(1/r), \tag{1.3}$$

where $-\alpha\le\theta\le\pi-\alpha$, $\langle\alpha\rangle$ is the discontinuity of α; σ and $C>0$ are given constants; λ is the minimum (within]–1; 0[) root of the equation

$$\Delta(-x-1)=0,\qquad \Delta(z)=b_0(z)+b_1(z)e+b_2(z)e^2,$$

$$b_0(z)=(\sin 2z\alpha+z\sin 2\alpha)\{(1+\chi_1)^2-4[\chi_1\sin^2 z(\pi-\alpha)+z^2\sin^2\alpha]\},$$

$$b_1(z)=(1+\chi_1)(1+\chi_2)\sin 2z\pi+4(\chi_2\sin 2z\alpha-z\sin 2\alpha)[\sin^2 z(\pi-\alpha)-z^2\sin^2\alpha]$$

$$-(\sin 2z\alpha+z\sin 2\alpha)\{(1+\chi_1)(1+\chi_2)-4[\chi_1\sin^2 z(\pi-\alpha)+z^2\sin^2\alpha]\},$$

$$b_2(z)=-4(\chi_2\sin 2z\alpha-z\sin 2\alpha)[\sin^2 z(\pi-\alpha)-z^2\sin^2\alpha],$$

$$\chi_{1,2}=3-4\nu_{1,2},\quad e=\frac{1+\nu_2}{1+\nu_1}e_0,\quad e_0=\frac{E_1}{E_2},$$

E_1 and E_2 are Young's moduli; ν_1 and ν_2 are Poisson's ratios; $g(\alpha, e_0, \nu_1, \nu_2)$ is a known function.

The values of λ for some values of α and e_0 and $\nu_1=\nu_2=0.3$ are summarized in Table 1.

The function g in (1.3) is expressed as

$$g=g_1/g_2,\quad g_1=K_1\cos\lambda\alpha+K_3\cos(\lambda+2)\alpha+[K_2\cos\lambda\alpha+K_4\cos(\lambda+2)\alpha]e,\quad g_2=K_1+K_2e,$$

$$K_1=(\lambda+2)[-2\sin(\lambda+2)\alpha\cos 2(\lambda+1)(\pi-\alpha)$$

$$+(1+\chi_1)\sin\lambda(2\pi-\alpha)+(\lambda+1)(\lambda+2)\sin(\lambda+2)\alpha\cos 2\alpha$$

$$-(\lambda+2)(2\lambda+1+\chi_1)\sin(\lambda+2)\alpha+\lambda(\lambda+1)\cos(\lambda+2)\alpha\sin 2\alpha+(\lambda+1)(\lambda+1+\chi_1)\sin\lambda\alpha],$$

$$K_2=2(\lambda+2)[\lambda(\lambda+2)-(\lambda+1)^2\cos 2\alpha+\cos 2(\lambda+1)(\pi-\alpha)]\sin(\lambda+2)\alpha,$$

$$K_3=-2(\lambda+1+\chi_1)\cos\lambda\alpha\sin 2(\lambda+1)(\pi-\alpha)+\lambda(1-\chi_1)\sin(\lambda+2)(2\pi-\alpha)$$