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ON THE DUGDAILL MODEL FOR A CRACK AT THE INTERFACE OF DIFFERENT MEDIA

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Within the framework of a plane static problem, we calculate the initial plastic zone near the end of a crack located at the interface of two different homogeneous isotropic media whose Young's moduli and Poisson coefficients are E_1, E_2 and ν_1, ν_2. Following the localization hypothesis [7], the initial plastic zone is modeled by a plastic band that starts from the end of the crack and corresponds to the Dugdaill model. It is assumed that the materials of the contacting bodies are considerably harder than the more plastic material of the binder (cement), owing to which the plastic band is also located at the interface of the media. The borders of the crack are free of stresses. Only a discontinuity of normal displacement is allowed in the plastic zone; the normal stress is equal to the tensile strength σ_s. Since the plastic-zone length is considerably smaller than the crack length L and all other dimensions of the bodies and the stress–strain state is studied only near the plastic zone, for the corresponding boundary-value problem we shall use the solution of the static problem for an elastic plane made up of two different half-planes containing at the interface of the bodies a semi-unbounded crack and a plastic zone (line) of length l that starts from the crack end. At infinity, the principal terms of the expansions of the stresses into asymptotic series are the asymptotically largest solution of the problem without a plastic zone that satisfies the condition of stress damping. This solution [6] is determined with accuracy to two arbitrary constants: the stress-intensity coefficients. These stress-intensity coefficients are given. They characterize the intensity of external field and are determined from the external-problem solution. The problem of the slip line at the end of a crack at the interface of different media was solved in [3] in a similar formulation.

The boundary conditions of the problem have the form

$$\theta = \pm\pi;\ \sigma_\theta = \tau_{r\theta} = 0; \quad \theta = 0;\ \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0; \quad \langle u_r \rangle = 0; \tag{1}$$

$$\theta = 0;\ r < l;\ \sigma_\theta = \sigma_s; \quad \theta = 0;\ r > l;\ \langle u_\theta \rangle = 0; \tag{2}$$

$$0 = 0;\ r \to \infty;\ \sigma_\theta = F(r) + \overline{F(r)} + o\left(\frac{1}{r}\right); \tag{3}$$

$$F(r) = e'(K_{\rm I} + iK_{\rm II})L^{-i\omega} r^{-1/2+i\omega};$$

$$e' = -\frac{1}{4}\frac{\kappa_1 + e + 1 + \kappa_2 e}{\sqrt{2\pi(\kappa_1 + e)(1 + \kappa_2 e)}}; \quad \omega = \frac{1}{2\pi}\ln\frac{\kappa_1 + e}{1 + \kappa_2 e};$$

$$\kappa_{1,2} = 3 - 4\nu_{1,2}; \quad e = \frac{E_1(1+\nu_2)}{E_2(1+\nu_1)}.$$

Here, r, θ is a polar coordinate system with the pole at the end of the crack and the polar axis directed along its continuation; σ_θ, $\tau_{r\theta}$ are stresses; u_θ, u_r are displacements; $\langle \alpha \rangle$ is a step of α; $\overline{W}$ is the complex conjugate of W; $K_{\rm I}$, $K_{\rm II}$ are given stress-intensity coefficients; and subscripts 1 and 2 correspond to half-planes $0 < \theta < \pi$ and $-\pi < \theta < 0$, respectively.

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Near the end of the plastic line, on the strength of general concepts of the behavior of stresses in the vicinities of nodes of elastic bodies [9, 11], an asymptotic form is realized that is the asymptotically largest solution, that satisfies the condition of displacement continuity, of the problem of elasticity theory for a piecewise-homogeneous plane that contains at the interface of the media a semi-unbounded cut, similar to the plastic line, with the corresponding homogeneous boundary conditions. This solution is

$$\sigma_\varphi = f(\rho)\left\{3(1+\kappa_2 e)\cos\frac{\varphi}{2} + [2(\kappa_1+e) - 1 - \kappa_2 e]\cos\frac{3\varphi}{2}\right\},$$

$$\tau_{\rho\varphi} = f(\rho)\left\{(1+\kappa_2 e)\sin\frac{\varphi}{2} + [2(\kappa_1+e) - 1 - \kappa_2 e]\sin\frac{3\varphi}{2}\right\},$$

$$\sigma_\rho = f(\rho)\left\{5(1+\kappa_2 e)\cos\frac{\varphi}{2} - [2(\kappa_1+e) - 1 - \kappa_2 e]\cos\frac{3\varphi}{2}\right\},$$

$$(0 \le \varphi \le \pi)$$

$$\sigma_\varphi = f(\rho)\left\{3(\kappa_1+e)\cos\frac{\varphi}{2} + [2(1+\kappa_2 e) - \kappa_1 - e]\cos\frac{3\varphi}{2}\right\},$$

$$\tau_{\rho\varphi} = f(\rho)\left\{(\kappa_1+e)\sin\frac{\varphi}{2} + [2(1+\kappa_2 e) - \kappa_1 - e]\sin\frac{3\varphi}{2}\right\},$$

$$\sigma_\rho = f(\rho)\left\{5(\kappa_1+e)\cos\frac{\varphi}{2} - [2(1+\kappa_2 e) - \kappa_1 - e]\cos\frac{3\varphi}{2}\right\},$$

$$(-\pi \le \varphi \le 0)$$

$$f(\rho) = \frac{K}{4(\kappa_1+e)\sqrt{2\pi\rho}}.$$

Here, ρ, φ is a polar coordinate system with the pole at the end of the cut and the polar axis directed along its continuation; $\sigma_\varphi, \tau_{\rho\varphi}, \sigma_\rho$ are stresses; and K is an arbitrary constant (stress-intensity coefficient). In particular, we have the asymptotic forms

$$\theta = 0,\ r \to l+0;\ \sigma_\theta \sim \frac{\kappa_1 + e + 1 + \kappa_2 e}{2(\kappa_1+e)} \frac{K}{\sqrt{2\pi(r-l)}};$$

$$\theta = 0,\ r \to l-0;\ \left\langle \frac{\partial u_\theta}{\partial r} \right\rangle \sim -\frac{4(1-\nu_1^2)}{E_1} \frac{1+\kappa_2 e}{1+\kappa_1} \frac{K}{\sqrt{2\pi(r-l)}}. \tag{4}$$

The stress-intensity coefficient K at the end of the plastic line is to be determined.

Thus, the introduction of a Dugdaill line at the end of the crack at the interface of two different media eliminates the oscillating singularity at the end of the crack.

The solution of the formulated boundary-value problem of elasticity theory with boundary conditions (1)–(3) is the sum of the solutions of the following two problems. The first differs in that the first condition of (2) is replaced by

$$\theta = 0,\ r < l,\ \sigma_\theta = \sigma_s - F(r) - \overline{F(r)}, \tag{5}$$

and at infinity the stresses are damped as $o(1/r)$. The second problem is similar but without a plastic line. Inasmuch as the solution of the second problem is known, it is sufficient to construct a solution of the first.

Applying the Mellin integral transform with the complex parameter p [10] to the equilibrium equations, the condition of deformation consistency, Hooke's law, and conditions (1) and allowing for the second condition of (2) and condition (5), we arrive at the Wiener–Hopf functional equation of the first problem