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STRESS STATE NEAR A SMALL-SCALE CRACK AT THE CORNER POINT OF THE INTERFACE OF MEDIA

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The stress state of a piecewise-homogeneous isotropic elastic body near a small-scale mode I crack at the corner point of the interface of media is analyzed. The exact solution of the corresponding elasticity problem is obtained using the Wiener–Hopf method.

Keywords: isotropic elastic body, interface, corner point, small-scale crack, Wiener–Hopf method

Introduction. The fracture of deformable solids begins usually near various corner points having the form of sharp stress concentrators. In the associated problem of linear elasticity, the stresses tend to infinity at such a corner point. The very high stress concentration at the corner point, discontinuity near it is possible and cracks much smaller than the body (small-scale cracks) can emerge from it. If the equilibrium of a crack is unstable, then will grow after the state of limit equilibrium is reached, which may result in collapse of the body. Therefore, the information on the stress state of an elastic body near cracks at the corner points is especially important for solving structural failure problems.

The stress state of elastic bodies at corner points in the vicinity of cracks is analyzed in many papers. Elastic problems on cracks and other discontinuity lines at the vertex of a homogeneous wedge were mainly solved [1, 5, 10, 14, 15, 17, 20, 24–27]. For piecewise-homogeneous bodies, cases where the corner point coincides with a crack tip were analyzed. They include elastic problems on lines of discontinuity at the tips of interphase cracks and cracks reaching the interface between media [3, 4, 6, 8]. The elastic problem on small-scale interphase shear cracks at the corner point of the interface of media is solved in [9].

In what follows, we will analyze the stress state of a piecewise-homogeneous isotropic linear elastic body near a small-scale mode I crack at the corner point of the interface of media.

1. Problem Statement. Basic Equations. Let us solve a plane static symmetric problem for a piecewise-homogeneous body with angle-shaped interface. The body is composed of isotropic elastic parts with Young's moduli E_1, E_2 and Poisson's ratios ν_1, ν_2 (see Fig. 1).

In view of the general principles on the behavior of stresses near corner points in elastic bodies [13], the corner point O of the interface is a stress concentrator with a power singularity. As $r \to 0$, the leading terms in the expansions of the stress into asymptotic series are solutions of the associated homogeneous elastic problem (problem K in Fig. 2) for a piecewise-homogeneous plane with angle-shaped interface generated by the unique root λ_0 on]–1; 0] of its characteristic equation

$$[\sin 2(\lambda+1)\alpha + (\lambda+1)\sin 2\alpha][æ_1 \sin 2(\lambda+1)(\pi-\alpha) + (\lambda+1)\sin 2\alpha]$$

$$+\Big\{(1+æ_1)(1+æ_2)\sin^2 \lambda\pi - [\sin 2(\lambda+1)\alpha + (\lambda+1)\sin 2\alpha]$$

$$\times [æ_1 \sin 2(\lambda+1)(\pi-\alpha) + (\lambda+1)\sin 2\alpha] - [\sin 2(\lambda+1)(\pi-\alpha) - (\lambda+1)\sin 2\alpha]$$

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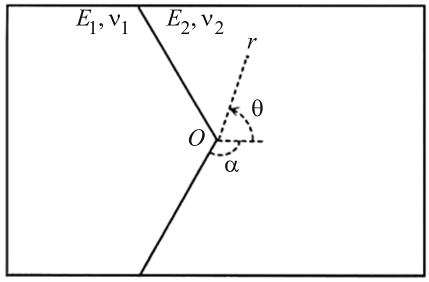

Fig. 1

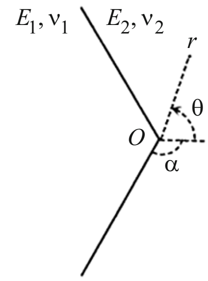

Fig. 2

$$\times\left[\text{æ}_2 \sin 2(\lambda+1)\alpha-(\lambda+1)\sin 2\alpha\right]\Big\}e$$

$$+[\sin 2(\lambda+1)(\pi-\alpha)-(\lambda+1)\sin 2\alpha][\text{æ}_2 \sin 2(\lambda+1)\alpha-(\lambda+1)\sin 2\alpha]e^2=0,$$

$$e=\frac{1+\nu_2}{1+\nu_1}e_0, \quad e_0=\frac{E_1}{E_2}, \quad \text{æ}_{1,2}=3-4\nu_{1,2}.$$

Wide classes of similar homogeneous elastic problems for wedge-shaped piecewise-homogeneous bodies that can be solved with the variable separation method are outlined in [19, 28].

The following formula holds:

$$\sigma_\theta(r,0)=Cgr^{\lambda_0}+o(r^{\lambda_0}) \quad (r\to 0), \qquad g=g_1+(\lambda_0+2)g_2,$$

$$g_1=(1+\text{æ}_2)\lambda_0^2 \sin 2\alpha \sin\lambda_0\alpha\cos\lambda_0\alpha\cos(\lambda_0+2)\alpha$$

$$-(1+\text{æ}_1)\lambda_0^2\sin 2\alpha\sin\lambda_0\alpha\cos\lambda_0(\pi-\alpha)\cos\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$-(1+\text{æ}_1)(1-\text{æ}_2)\lambda_0\sin\lambda_0\alpha\cos\lambda_0\alpha\sin(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\cos\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$-(1-\text{æ}_1)(1+\text{æ}_2)\lambda_0\sin\lambda_0\alpha\cos\lambda_0\alpha\cos(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\sin\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$-(1-e)(\lambda_0+1-\text{æ}_2)\lambda_0^2\sin^2 2\alpha\cos\lambda_0\alpha$$

$$-(2e-1+\text{æ}_1)\lambda_0(\lambda_0+1-\text{æ}_2)\sin 2\alpha\cos\lambda_0\alpha\cos\lambda_0(\pi-\alpha)\sin\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$-[2-e(1-\text{æ}_2)]\lambda_0(\lambda_0+1-\text{æ}_2)\sin 2\alpha\cos^2\lambda_0\alpha\sin(\lambda_0+2)\alpha$$

$$-2[(1-\text{æ}_2)e-1+\text{æ}_1](\lambda_0+1-\text{æ}_2)\cos^2\lambda_0\alpha\sin(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\sin[\lambda_0(\pi-\alpha)-2\alpha],$$

$$g_2=(1+\text{æ}_1)\lambda_0\sin 2\alpha\sin(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\cos[\lambda_0(\pi-\alpha)-2\alpha]$$

$$+(1+\text{æ}_1)(1-\text{æ}_2)\cos\lambda_0\alpha\sin^2(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\cos\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$-(1+\text{æ}_2)\lambda_0\sin 2\alpha\cos\lambda_0\alpha\sin(\lambda_0+2)\alpha\cos(\lambda_0+2)\alpha$$

$$+(1-\text{æ}_1)(1+\text{æ}_2)\cos\lambda_0\alpha\sin(\lambda_0+2)\alpha\cos(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\sin\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$

$$+(1-e)\lambda_0^2\sin^2 2\alpha\cos(\lambda_0+2)\alpha$$

$$+(2e-1+\text{æ}_1)\lambda_0\sin 2\alpha\cos(\lambda_0+2)\alpha\cos\lambda_0(\pi-\alpha)\sin\left[\lambda_0(\pi-\alpha)-2\alpha\right]$$