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# COTTRELL CRACK NUCLEATION CONDITION

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**The length of the small-scale narrow zone of weakened bonds (fracture process zone) at the point of intersection of lines of microplastic deformation (slip lines) in an elastic body is determined. The fracture process zone is modeled by a line of discontinuity of normal displacement. The development of this zone precedes Cottrell crack nucleation. The crack nucleation condition is established. The Wiener–Hopf method is used to obtain the exact solution of the associated elastic problem.**

**Keywords:** fracture process zone, Cottrell crack nucleation, Wiener–Hopf method

**Introduction.** A significant number of publications are devoted to plane problems of fracture mechanics solved to determine the fracture process zones near crack tips and other corner points such ones as sharp stress concentrators in homogeneous bodies within the framework of models with lines of displacement discontinuity. Such modeling is based on the classical experimental and theoretical studies [2, 5, 7, 12], which show that initial fracture process zones are thin layers in the form of narrow stripes emerging from sharp stress concentrators. The model of a plastic fracture process zone with two lateral lines of discontinuity of tangential displacement (slip lines) [9, 13, 18–21] is widely used in elastoplastic plane problems of fracture mechanics. This model is also employed in determining an initial plastic zone near the end of a thin rigid inclusion [1] and close to the corner point of the body boundary [4]. In the case of elastic body, the Leonov–Panasyuk model [7] is used, according to which a narrow zone of weakened bonds modeled by a line of normal displacement discontinuity arises and develops on the continuation of a mode I crack. The normal stress on this line is equal to a given material constant (rupture strength).

The point of intersection of the lines of discontinuity of the tangential displacement (slip lines) is a sharp stress concentrator with power singularity (Fig. 1). The problems devoted to determining fracture process zones near this point within the framework of the above models are not considered. The solutions of such problems can be used in studying one of the several dislocation mechanisms of crack nucleation, namely Cottrell's mechanism [8].

There are experimental data that show that microplastic deformation consisting in motion and retardation of dislocations and resulting in crack nucleation precedes the brittle fracture of the body.

In accordance with Cottrell's mechanism, the coalescence of two dislocations moving in intersected slip planes gives rise to a sessile dislocation that hinders the motion of other dislocations and leads to crack nucleation. Dislocations moving in both slip planes stop before this obstacle, producing a dislocation pileup that causes high stress concentration near the obstacle. Such stress concentration causes the loss of integrity and origin of cracks.

The development of a narrow fracture process zone with weakened bonds near the intersection point of lines of microplastic deformation precedes the crack nucleation by Cottrell's mechanism. A crack can be expected to arise along the weakened-bond zone at a certain level of the external load.

In what follows, we will establish the condition for the nucleation of a Cottrell crack by analyzing the above zone within the framework of the Leonov–Panasyuk model.

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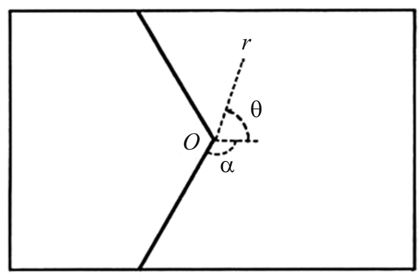


Fig. 1

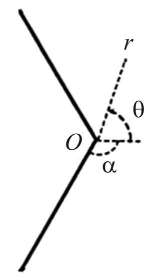


Fig. 2

TABLE 1

| α, deg. | $-\lambda_0$ | Λ | Δ |
|---|---|---|---|
| 100 | 0.190 | 4.799 | 1.599 |
| 110 | 0.335 | 11.648 | 3.882 |
| 120 | 0.449 | 10.616 | 3.925 |
| 130 | 0.541 | 8.759 | 3.105 |
| 140 | 0.619 | 7.022 | 2.341 |
| 150 | 0.689 | 5.502 | 1.834 |
| 160 | 0.756 | 4.163 | 1.387 |
| 170 | 0.831 | 2.872 | 0.957 |

**1. Problem Statement.** Let us consider an isotropic elastic body under plane strain conditions within the framework of a static symmetric problem. The body contains intersecting lines of microplastic deformation (Fig. 1, where $\pi/2 < \alpha < \pi$). We will model the line of microplastic deformation by a line of discontinuity of the tangential displacement on which the tangential stress is equal to the material constant $\tau_s$ characterizing the microplastic deformation of the elastic body ($\tau_s$ is the shear yield stress).

In view of the general principles of the behavior of stresses near the corner points of elastic bodies [10], the point $O$ is a sharp stress concentrator with power singularity. As $r \to 0$, the sums of the leading terms of asymptotic series expansion of the stresses are the solution of the appropriate elasticity problem (problem K, Fig. 2) for a plane with semi-infinite lines of discontinuity. The solution is attributed to the root $\lambda_0 \in ]-1, 0]$ (which is unique in the band $-1 < \operatorname{Re}\lambda < 0$) of the characteristic equation

$$\left[\cos 2\alpha - \cos 2(\lambda+1)\alpha\right]\left[\sin 2(\lambda+1)(\pi-\alpha) - (\lambda+1)\sin 2\alpha\right]$$

$$+\left[\cos 2\alpha - \cos 2(\lambda+1)(\pi-\alpha)\right]\left[\sin 2(\lambda+1)\alpha + (\lambda+1)\sin 2\alpha\right] = 0.$$

Particularly, we have:

$$\sigma_\theta(r,\theta) = C\Sigma(\theta) r^{\lambda_0} + C_0 + \frac{\tau_s}{\sin 2\alpha}\cos 2\theta + f(r,\theta) \quad (r \to 0),$$