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INITIAL FRACTURE PROCESS ZONE AT THE CORNER POINT OF THE INTERFACE BETWEEN ELASTIC BODIES

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A symmetric problem of elasticity is solved to determine the initial fracture process zone at the corner point of a V-shaped interface in a piecewise-homogeneous isotropic body. The interface is a thin elastic layer joining two isotropic elastic materials. The initial fracture process zone is modeled by lines of discontinuity of normal displacement located at the interface. The exact solution of the problem of linear elasticity is found with the Wiener–Hopf method and then used to determine the length of the fracture process zone. The stress state near the corner point is examined

Keywords: fracture process zone, corner point, interface, lines of discontinuity of normal displacement, Wiener–Hopf method

Introduction. In solving plane problems to determine narrow fracture process zones near crack tips and corner points (sharp-pointed stress concentrators) in homogeneous and piecewise-homogeneous bodies, models with discontinuity lines of displacements are widely used [1, 3, 4, 6, 8, 10, 11, 14, 19]. For example, the plastic fracture process zone in elastoplastic plane-strain problems of fracture mechanics is modeled by two lateral lines of discontinuity of tangential displacement (slip lines) [10, 14]. The fracture process zone (zone of weakened bonds) near a crack tip in an elastic body is modeled by a line of discontinuity of normal displacement [6, 8]. The existence of such zones (craze cracks) in polymers was confirmed by experiments [2].

Determining the size of fracture process zones near sharp-pointed stress concentrators in a piecewise-homogeneous body with a thin bonding layer as an interface is of considerable interest for the fracture mechanics of adhesive joints. The plastic fracture process zone located near a corner point of the elastoplastic interlayer (glue) between two elastic materials and propagating along the interface was examined in [18]. The present paper addresses the case where the bonding layer is elastic (embrittled glue), i.e., the initial stage of debonding in a piecewise-homogeneous isotropic elastic body along the interface near the corner point.

1. Problem Statement. Consider a piecewise-homogeneous isotropic elastic body with a V-shaped interface in symmetric plane-strain conditions. The interface is a thin elastic bonding layer (embrittled glue) that is less crack-resistant than the bonded materials.

As the external load is increased, a narrow fracture process zone (zone of weakened bonds), which is a sharp-pointed stress concentrator much smaller then the body, forms and grows. Let us examine only the initial stage of evolution of this zone.

Let in the problem of elasticity representing the stage at which the fracture process zone is yet absent (problem A), the normal stresses near the corner point at the interface be tensile (condition T). The associated constraints for the problem parameters will be given below. The initial fracture process zone has the form of two narrow bands emerging from the corner point and located at the interface.

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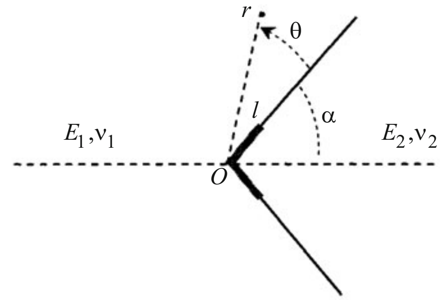

Fig. 1

This is the initial stage of debonding in the piecewise-homogeneous body along the interface near its corner point. Therefore, the tasks is to determined the length of the fracture process zone and to analyze the stress state near the corner point.

Since the bonding material is elastic, the predominant strains in the fracture process zone are tensile, and band is modeled by a discontinuity line of normal displacement on which the normal stress is equal to a given constant of the bonding material σ (rupture strength) [8].

Since the fracture process zone is small, it can be determined by solving a plane static symmetric problem of linear elasticity for a piecewise-homogeneous isotropic plane with a V-shaped interface containing finite-length cuts emerging from the corner point (Fig. 1). At infinity, the solution asymptotically tends to that of the problem without cuts (problem K) generated by the unique (on]–1, 0[) root of its characteristic equation. Let the arbitrary constant C that appears in this solution be given. It characterizes the intensity of the external field and is determined by solving the external problem [14, 18].

The boundary conditions of the problem (Fig. 1) are the following:

$$\theta=\pi-\alpha,\quad \tau_{r\theta}=0,\quad u_\theta=0,\qquad \theta=-\alpha,\quad \tau_{r\theta}=0,\quad u_\theta=0, \tag{1.1}$$

$$\theta=0,\quad \langle\sigma_\theta\rangle=\langle\tau_{r\theta}\rangle=0,\quad \langle u_r\rangle=0,$$

$$\theta=0,\quad r<l,\quad \sigma_\theta=\sigma,\quad \theta=0,\quad r>l,\quad \langle u_\theta\rangle=0, \tag{1.2}$$

$$\theta=0,\quad r\to\infty,\quad \sigma_\theta=Cgr^{\lambda_0}+o\left(\frac{1}{r}\right), \tag{1.3}$$

where $-\alpha\le\theta\le\pi-\alpha$; $\langle a\rangle$ is the discontinuity of a; λ_0 is the root (unique on]–1, 0[) of the equation

$$\Delta(-\lambda-1)=0,\quad \Delta(z)=\delta_0(z)+\delta_1(z)e+\delta_2(z)e^2$$

$$\Big(\delta_0(z)=(\sin 2z\alpha+z\sin 2\alpha)[\varkappa_1\sin 2z(\pi-\alpha)+z\sin 2\alpha],$$

$$\delta_1(z)=(1+\varkappa_1)(1+\varkappa_2)\sin^2 z\pi-(\sin 2z\alpha+z\sin 2\alpha)$$

$$\times[\varkappa_1\sin 2z(\pi-\alpha)+z\sin 2\alpha]-[\sin 2z(\pi-\alpha)-z\sin 2\alpha](\varkappa_2\sin 2z\alpha-z\sin 2\alpha),$$

$$\delta_2(z)=[\sin 2z(\pi-\alpha)-z\sin 2\alpha](\varkappa_2\sin 2z\alpha-z\sin 2\alpha),$$

$$e=e_0(1+\nu_2)/(1+\nu_1),\quad e_0=\frac{E_1}{E_2},\quad \varkappa_{1,2}=3-4\nu_{1,2},$$

E_1, E_2 are Young's moduli; ν_1,ν_2 are Poisson's ratios; g is expressed as

$$g=g_1\cos\lambda_0\alpha+g_2\cos(\lambda_0+2)\alpha$$

$$\Big(g_1=[(\lambda_0+2)e-\lambda_0-2][(\lambda_0+2)\cos\lambda_0(\pi-\alpha)\sin(\lambda_0+2)(\pi-\alpha)$$

$$-\lambda_0\sin\lambda_0(\pi-\alpha)\cos(\lambda_0+2)(\pi-\alpha)]\cos(\lambda_0+2)\alpha$$