Reinforced Concrete Rod Elements Stiffness Considering Concrete Nonlinear Properties

T. Azizov[1], D. Kochkarev[2], and T. Galinska[3](✉)

[1] Pavlo Tychyna Uman State Pedagogical University, Uman, Ukraine
[2] National University of Water and Environment Engineering, Rivne, Ukraine
[3] Poltava National Technical Yuri Kondratyuk University, Poltava, Ukraine
galinska@i.ua

Abstract. In this paper results of stiffness iterative determination technique while torsion the reinforced concrete rod elements considering concrete nonlinear properties is given. The section of the element is divided into several rectangular elements. The shearing torsions at each iteration step are determined by the distribution of torsions, which magnitude is higher than the limit, over the elements where the torsions are in the elastic area. An algorithm for obtaining points on the "Torque moment is relative twisting angle" diagram of a reinforced concrete rod elements is given. Its advantage from a similar technique is that shear torsions are determined directly from the concrete shear diagram, and not according to the formulas of the elasticity theory. The above algorithm enables to calculate reinforced concrete rod elements using any concrete shear diagrams obtained both experimentally and theoretically, as well as elements of any cross-section. Limiting torsions are determined directly from the concrete shear diagram.

Keywords: Torsion · Reinforced concrete rod · Nonlinear properties · Diagram · Iteration

1 Problem Statement and Research Analysis

Quite a lot of work has been devoted to the calculation of reinforced concrete elements. However, the calculation of the strength and stiffness of reinforced concrete elements spatial sections and in the case of cracking, spatial torsion cracks are considered in most works (Krainskyi et al. 2018; Khmil et al. 2018; Ponamonov 2012; Karpenko 1976; Cowan 1972). Investigations concerning the calculation of reinforced concrete elements with normal cracks in torsion have been carried out in the works (Azizov 2009; Sribyanik 2009; Azizov 2007). Mainly the problem of determining the torsion state of a block separated by normal cracks, which torque moments are transmitted via part of the section is solved in these works.

An approximate solution of reinforced concrete rod elements torsion problem considering nonlinear properties was studied in (Kochkarev et al. 2018; Jurkowska 2018; Jurkowska 2019; Yaremenko and Shkola 2010; Karpenko 1976). Nonlinear properties of concrete were taken into consideration. However, in this paper,

Z. Blikharskyy et al. (Eds.): CEE 2019, LNCE 47, pp. 1–6, 2020.
https://doi.org/10.1007/978-3-030-27011-7_1

a significant error was made when the tangential torsions in the cross-section were determined by the theory of elasticity formulas. It cannot be fallen short of expectations.

Taking into account the above mentioned, the purpose of this article is to develop a method for determining the torsion stiffness of reinforced concrete rod element considering nonlinear concrete properties.

2 Main Material Presentation

Consider the typical section of the rod element subjected to torsion. For the clarification simplicity, it will be considered a section without reinforcement. Considering reinforcement is carried out like in the calculation of reinforced concrete elements for bending by multiplying the area of reinforcement elements by a modular ratio equal to the ratio of the reinforcement elastic modulus to the elastic modulus of concrete. Besides for clarification simplicity, it will be considered a rectangular section. Any other section elements can be calculated using the method described below.

The cross-section is divided into several rectangles (Fig. 1).

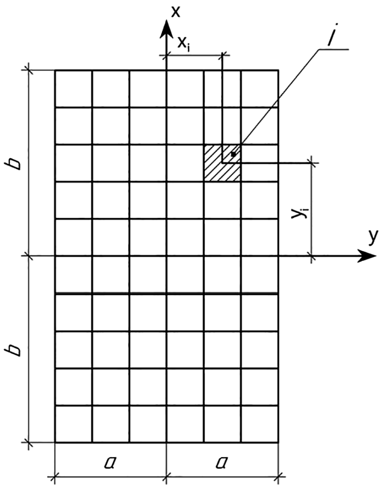

Fig. 1. Cross section dividing into separate elements diagram

Draw the X and Y axes from the element center of torsion, which in the general case may not coincide with the center of the section gravity (Harutyunyan 1963). The definition of a torsion center is not difficult. Each i element (shaded in Fig. 1) has coordinates Xi and Yi. Each element has tangential forces:

$$T_{yz,i} = \tau_{yz,i} A_i ; T_{xz,i} = \tau_{xz,i} A_i . \tag{1}$$

If the distribution of tangential torsions over the cross section is known $\tau_{yz,i}$ and $\tau_{xz,i}$, it is not difficult to calculate the tangential forces $T_{xz,i}$ и $T_{yz,i}$, and, therefore, the torque in cross section:

$$M_t = \sum_{i=1}^{n} \left(T_{yz,i} X_i + T_{xz,i} Y_i \right). \tag{2}$$

Where n is the quantity of the total elements, the section is broken into.

From the theory of torsion, it is known (Harutyunyan 1963) that the torque moment Mt in cross section is associated with the relative angle of twist Θ dependence:

$$M_t = D \cdot \theta. \tag{3}$$

Where D is rod stiffness (in the section under consideration) in torsion.

It should be particularly noted that the dependence (Azizov 2010) is valid for any distribution of torsions in the element cross section. In other words, if the distribution of tensions in the section (and the torque moment in the section), as well as the relative twisting angle, is known, then the stiffness of section D is certainly determined from the expression (Azizov 2010). And vice versa, if stiffness D is known, then the relative twist angle is certainly determined from (Azizov 2010). This is a significant point to consider. It also helps to determine the stiffness of the element during torsion.

Consider the algorithm for determining the stiffness in the cross section of the rod element considering concrete nonlinear properties. In this case, first, the shear diagram will be taken in the form of the Prandtl diagram. Such a presentation of the diagram is based on experimental studies conducted in (Vildanova 2015).

1. Set the preliminary value of the twist relative angle θ1;
2. Assuming elastic rod work, determine the elastic moment $M_{t,e}$, which corresponds to a given twist angle:

$$M_{t,e} = GI \cdot \theta_1. \tag{4}$$

   Where GJ is torsion stiffness of the rod assuming its elastic work;
3. According to the theory of elasticity formulas (for example, according to [9]), determine the tangential torsions $\tau_{yz,i}$ and $\tau_{xz,i}$ in the center of every i- element;
4. Determine the maximum stress value. $\tau_{yz,max}$, and $\tau_{xz,max}$
5. Determine the ratio

$$k_{yz,i} = \tau_{yz,i} / \tau_{yz,max}; k_{xz,i} = \tau_{xz,i} / \tau_{xz,max}. \tag{5}$$

6. Calculate all ratio sums. $k_{yz,i}$ and $k_{xz,i}$, which values are less than one:

$$k_{tot,x} = \sum_{j=1}^{m} k_{xz,j}; k_{tot,y} = \sum_{j=1}^{n} k_{yz,j}. \tag{6}$$

   Where n, m is elements quantity where the stress exceeds the limit, respectively $\tau_{yz,i}$ and $\tau_{xz,i}$.

7. If in any elements, the shear torsions are greater than the limit shear torsions $\tau i > [\tau]$, then the stress in this element is taken equal to $[\tau]$. It is considered as a stress τ_{yz}, and as τ_{xz};
8. Calculate the value of the "extra" moments in each element, where the torsions exceed the limiting:

$$\varDelta M_{y,i} = \left(\tau_{yz,max} - \tau_{yz,i}\right)A_i \cdot X_i; \varDelta M_{x,i} = \left(\tau_{xz,max} - \tau_{xz,i}\right)A_i \cdot Y_i. \tag{7}$$

9. Calculate the total "extra" torque moments

$$M_{e,y} = \sum_{k=1}^{n} \varDelta M_{y,k};\ M_{e,x} = \sum_{k=1}^{m} \varDelta M_{x,k}. \tag{8}$$

   Where m and n are the same as in point 6.
10. We distribute the "extra" moments between the elements where $k_{yz,i}$, and $k_{xz,i}$ are less than one, in proportion to the ratio:

$$M_{y,i} = \frac{M_{e,y}}{k_{tot,y}} k_{yz,i}; M_{x,i} = \frac{M_{e,x}}{k_{tot,x}} k_{xz,i}. \tag{9}$$

11. Determine the new values of the tangential torsions in the elements where, at the first iteration, the torsions were less than the limiting ones (taking into account the added "extra" torsions:

$$\tau^r_{xz,i} = \tau_{xz,i} + \frac{M_{x,i}}{A_i Y_i}; \tau^r_{yz,i} = \tau_{yz,i} + \frac{M_{y,i}}{A_i X_i}. \tag{10}$$

12. Further, the calculation is repeated, starting with point 6. In this case, the elements quantity m and n, where the torsions exceed the permitted ones, will already differ from the previous ones.

Note: at each iteration step, the position of the torsion center should be specified. Accordingly, Xi and Yi also are specified.

Thus, at the end of the iterative calculation, obtain the final distribution of tangential torsions.

13. Knowing the final torsion values $\tau_{xz,i}$ and $\tau_{yz,i}$ in all elements the cross section is divided into(see Fig. 1), using Eq. (1), calculate the tangential forces $T_{yz,i}$ and $T_{xz,i}$;
14. By the Eq. (2) determine the torque moment Mt from intensification $T_{yz,i}$ and $T_{xz,i}$. At that time, the torque moment Mt will be different from the first moment $M_{t,e}$, determined by Eq. (4). This is because some torsions exceeding the value $[\tau]$, will no longer contribute to the total torque moment.

So we get the torque value Mt, corresponding to the relative twist angle given in point 1 θ 1 or the first point on the diagram «Mt – θ».

15. Next, increase the value of the relative twist angle θ2 and, after performing the calculation using the above algorithm, obtain the second point in the diagram.

Continuing incrementally increasing the values of the twist angle, get the diagram «Mt – θ», it is easy to obtain the values of the twisting angle with the real torque moment acting on the real rod, as well as the value of the torsion stiffness D corresponding to the specified real torque moment, since knowing θ and Mt, by the Eq. (3) the stiffness D is easy to determine.

If at the initial stages of calculation, it turns out that all torsions are less than the limit, and then this part of the diagram «Mt – θ» is built on points considering elastic laws.

It should be mentioned as a significant remark. If the real diagram of the concrete shear is known (Vildanova 2015), then the torsions in the elements according to the point of the algorithm should be taken directly from the diagram. As mentioned above, expression (Azizov 2007) is valid for any distribution of tangential torsions in the section. So, to set the specific value of the twisting angle, then, using the concrete shear diagram, determine all torsions (exceeding elastic values) and by the formula (Azizov 2010) taking into account (Azizov 2009) determine the moment Mt. And get the point on the diagram «Mt – θ». Further, similarly to the above, we repeat the calculations with other values of the twist angle and get the diagram «Mt – θ».

Rod bearing capacity exhaustion will be the top point in the diagram.«Mt – θ», when the value of torque moment is reduced because in separate elements there will be deformations equal to the limit deformation in the concrete shear diagram.

According to the described algorithm, it is easy to get a diagram.«Mt – θ» for rods with a section differing from a rectangular section. The principle of calculation does not change for any section. There are solutions to the theory of elasticity by definition of tangential torsions.

To calculate a rod with a cross section which has no solutions to the theory of elasticity by definition of tangential torsions, the following technique can be used. Similar to (Azizov 2010), based on calculations using well-known programs like Ansys, Lira, and others, using finite bulk elements, obtain the laws of the distribution of tangential torsions over the cross-section. Then the calculation considering the concrete nonlinear properties is carried out according to the algorithm described above.

Longitudinal reinforcement are not marked in Fig. 1. The presence of reinforcement is easy to take into account (while calculating reinforced concrete elements by bending) by replacing its cross-section with an equivalent by multiplying its cross-sectional area by reinforcement and concrete elastic modulus ratio.

In the case of normal cracks as it follows from the diagram in Fig. 1 exclude concrete elements which are within the crack. In this case, the determination of the gravity center and torsion center in a cross-section is easy to perform in resistance to materials known method.

3 Conclusions and Research Prospects

An algorithm for obtaining points on the "Torque moment is relative twisting angle" diagram of a reinforced concrete rod element is given. Its advantage from a similar technique is that shear torsions are determined directly from the concrete shear diagram, and not according to the formulas of the elasticity theory. The above algorithm enables to calculate reinforced concrete rod elements using any concrete shear diagrams obtained both experimentally and theoretically, as well as elements of any cross-section.

In the future, it is planned to develop a computer program and experimental verification of the developed methodology.

References

Azizov T (2010) The use of approximation finite elements in the calculations of structures. News Odessa State Power Acad Archit 39(1):4–9

Azizov TN (2009) Stiffness of reinforced concrete elements in torsion and its influence on the spatial work of bridges NAS of Ukraine. Physico-mech. Institute of Im .. Karpenka, VG, pp 576–590

Cowan GJ (1972) Torsion in ordinary and prestressed concrete: Trans. from English Publishing house of literature on construction, p 104

Harutyunyan NKh (1963) Torsion of elastic bodies. Publishing house of physical and mathematical literature, p 686

Jurkowska N (2018) Considering nonlinear properties of concrete in the design of reinforced concrete structures for torsion. Paper presented at the IOP conference series: materials science and engineering, vol 364, no (1). https://doi.org/10.1088/1757-899x/364/1/012030

Jurkowska N (2019) Presentation of the exact technique for calculation of the torsional constant for the T-section. Paper presented at the IOP conference series: materials science and engineering, vol 473, no (1). https://doi.org/10.1088/1757-899x/473/1/012034

Karpenko NI (1976) The theory of reinforced concrete deformation with cracks. Stroiizdat, p 208

Khmil R, Tytarenko R, Blikharskyy Y, Vegera P, (2018) Development of the procedure for the estimation of reliability of reinforced concrete beams, strengthened by building up the stretched reinforcing bars under load, Eastern-Eur J Enterp Technol 5/7(95)

Kochkarev D, Azizov T, Galinska T (2018) Bending deflection reinforced concrete elements determination. Paper presented at the MATEC Web of conferences, p 230. https://doi.org/10.1051/matecconf/201823002012

Krainskyi P, Blikharskyy Y, Khmil R, Vegera P (2018) Influence of loading level on the bearing capacity of RC columns strengthened by jacketing. Matec web of conferences, p 230

Paramonov DJu (2012) Rigidity and strength of reinforced concrete elements with normal cracking when torsion-bending, p 290

Sribyanik NM (2009) Rotational rigidity of reinforced concrete elements of overlappings with normal cracks: author’s abstract. dis … Cand. tech Sciences 05.23.01; Odessa State Academy of Civil Engineering and Architecture, p 23

Vildanova NR (2015) Modulus of shear of concrete in view of plasticity deformations and its influence on torsional stiffness of reinforced concrete elements: dis. … candidate of technical sciences … 05.23.01 « Building constructions, buildings and structures » , p 214

Yaremenko O, Shkola Yu, (2010) Bearing ability and deformability of reinforced concrete rod elements in a complex stressed state. ODABA, p 136

Azizov T (2007) Determination of reinforced concrete elements torsional stiffness with cracks. Roads and bridges. Scientific Paper Collection, vol 7, no (1), pp 3–8