Using of augmented reality technology in the process of training future teachers during the study of mathematical disciplines

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Abstract

Digital technologies, in particular augmented reality (AR), are rapidly changing the field of education, especially in science, technology, art and mathematics. The Mathematical Analysis course is one of the largest courses that provide fundamental training for future teachers of mathematics, physics and computer science. One of the successful components of the quality learning material and its further use in the learning process is visualization. The article analyzes usage of GeoGebra AR in solving problems on "Geometric and mechanical applications of integrals" (one function and several variables), "Area of definition of the function of several variables" course "Mathematical Analysis" in training future teachers of mathematics, physics and computer science. The efficiency and expediency of using practical tools based on GeoGebra AR for solving problems of the course "Mathematical Analysis" highlighted. The advantages of the dynamic mathematical system GeoGebra AR highlighted. The examples show the tasks, using new technologies that improve the process of teaching and learning mathematics. When solving problems, future teachers of mathematics, physics and computer science should develop the following components: a dynamic model of the body given by the condition of the problem in GeoGebra AR, videos; solving problems in the GeoGebra environment.

Keywords AR -technology, education, technology, maths, innovation, visualization of mathematical objects future teachers.

1. Introduction

1.1.The urgency of the problem.

The rapid development of the modern information society is inextricably linked to the flexible renewal and intensive rethinking of the university education system. The quality and effectiveness of modern university education is based on the deep immersion of both students and teachers in the digital information environment. Digital technologies and their applications in the form of innovative developments have become a part of our present. Technology will not disappear from our lives, it will penetrate even more into all spheres of human activity, life and everyday life. The digital giants of the world specializing in information technology offer a variety of concepts for human development in the short term. In fact, preparing humanity for a new era of development - Intelligent World. According to David Wang, CEO and President of Huawei's ICT Products and Solutions Division, by 2030, the integration of the digital and physical worlds will deepen significantly, creating a space as close as possible to the real world. The digital economy will be the main driver of all sectors of the economy.

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The modern education system faces challenges it did not know before. Mobile applications, online classes, the availability of modern digital skills have become part of everyday life and radically change the learning process. Education must respond as much as possible to the global challenges of digital transformation and key scientific and technological trends to use them, moving resolutely towards a world of barrier-free communication.

Improving the digital competence of teachers and students is aimed at finding, analyzing, critically evaluating and managing digital content; use digital devices and technologies to create knowledge and innovation in educational spaces; collaborate and learn with digital technologies and devices; be able to protect their work, taking into account of reliability and confidentiality; creatively usage of digital technologies to solve educational, technical, scientific problems; to be constantly engaged in the process of self-improvement and self-development; it is reasonable to avoid the risks and dangers that accompany the irrational use of new technologies [9]. According to the use of modern digital technologies in the formation of digital competence of modern students are closely related processes of knowledge acquisition, new competencies based on the latest tools, one of which is the means of augmented and virtual reality.

1.2.Analysis of recent research and publications.

The modern digital technologies, such as augmented reality (AR), virtual reality (VR) and 3D printers, provide new opportunities for education. Each of these technologies has a positive effect on learning. The interest in these technologies is growing among teachers, but not all teachers have the appropriate skills. This creates certain barriers in educational communication. Augmented reality as a form of the virtual world is a living, direct or indirect physical representation that adds elements around the real world of people. Applicants at different levels are increasingly willing to interact with subjects in cyberspace, which can lead to the expansion of educational activities, improving existing and new organizational forms, types and methods of teaching, improving student interaction and educational space [13]. Augmented Reality (AR) is a new technology that is changing education and displacing the game as an educational technology [1]. Theoretical analysis of recent research in this area justifies the need and feasibility of using the above approach in the training of students, and augmented reality (AR) technologies create unique opportunities in education.

Features of the use of augmented and virtual reality, their impact and importance in the educational process of various specialties and areas were covered in articles by domestic and foreign scientists [2, 3, 4, 6, 11, 12, 18, 20, 22, 23].

The Scientists Lutvunova, S., Burov, O., & Semerikov, substantiated the principles (expediency, accessibility, cognition, integrity, educational orientation, mobility) and approaches (cognitive, systemic, activity, differentiated, personality-oriented, innovative), described the pedagogical conditions and the advantages and disadvantages of using AR-technology in the educational process are specified [17].

Researchers believe that augmented and virtual reality technologies make the discipline more interesting. This facilitates the learning process. Modern e-learning technologies have many advantages, but they are gradually reducing the share of direct dialogue between teacher and student. The teacher is replaced by interactive content, and the student instead of a person interacts with his simplified virtual model. The introduction of VR-content in education gives positive results when used in short classes or as simulators. It is not advisable to use VR for lectures and seminars [14].

Peculiarities of the use of virtual and augmented reality in the study of mathematical disciplines have been studied by the following domestic and foreign scientists [9, 10, 15].

While learning mathematics, students often find it difficult or lose interest due to the complexity of the subject. Augmented reality (AR) could aid students to learn certain concepts or theories that were difficult to understand in the classroom and also has the capability to enhance the instructional/learning material [16].

Researchers believe that using AR in the study of mathematical disciplines, you can build a visual model of educational material, supplementing it with appropriate visual information; to achieve the

development of students' spatial imagination, which enhances their deep understanding of processes, properties, proof of theorems, etc. The use of AR technology allows to radically change the method of teaching subjects, giving students the opportunity to become direct participants in research processes in real time, without being distracted by routine mechanical calculations [9].

In particular, the problems of implementing augmented reality GeoGebra 3D Graphing Calculator with AR in the study of geometry are devoted to articles [5, 15, 21]

Because augmented reality is significantly related to 3D constructions, its use is possible with dynamic mathematics systems, including GeoGebra. This can significantly raise the level of visualization in the study of mathematical disciplines and enhance the learning of pupils and students. In addition, AR can be a tool to enhance STEM training for students and future teachers of physics, mathematics and computer science.

Among [21] the main advantages and features of using the most popular augmented reality applications (AR Ruler, AR Physics, Nicola Tesla, Arloon Geometry, AR Geometry, GeoGebra 3D Graphing Calculator) in university training are substantiated and revealed.

The article [19] discusses cross-platform products that should be used to develop augmented reality technologies: Unreal Development, Kit, Unity, Godot, Engine, Cocos2D, MonoGame, Unreal Engine, Marmalade, and the capabilities of known SDKs for augmented reality applications (Wikitude, Vuforia, Kudan, Maxst, Xzimg, NyARToolkit, Metaio SDK).

The aim of the study is to consider the features of the use of augmented reality in the study of mathematical disciplines.

The research used methods: analysis of scientific and pedagogical literature on the choice of theoretical and methodological foundations of augmented reality in the learning process at the university, pedagogical experiment to improve the effectiveness of the mobile application AR in teaching.

2. Results of the research

The educational sphere faces challenges that it did not know before. Mobile applications, online classes and augmented and virtual reality have become a part of everyday life and are radically changing the learning process. After all, these technologies allow all participants of the educational process to better perceive complex information and gain new skills, present information in an interactive form, demonstrate and apply theory during the lesson and understand how to use this knowledge in practice. Augmented reality technology is an evolving technology. But it already has a number of applications designed for education and training. More and more mathematical environments are complementing their functions with augmented reality tools. The use of such an ICT tool in the study of new material provides an opportunity to improve students' spatial imagination, “see” and better understand the learning material, which will contribute to its better assimilation and the formation of certain practical skills. 3D images are reproduced using a mobile phone, which is an integral attribute of the modern student [8].

The main advantages of AR-technology include: clarity, security, interactivity, integration with traditional methods. However, this technology has a number of disadvantages: the amount of content to fill the discipline, functionality and high cost of equipment, lack of high-quality applications and, consequently, the need to develop them, little experience in using this technology from teachers who need additional training and health.

This technology is effective in students’ university studying subjects such as “Linear Algebra”, “Analytical Geometry”, “Mathematical Analysis”, “Probability Theory and Mathematical Statistics” and more. Traditional 2D constructions on paper do not give a complete picture of the spatial configuration of the figures.

Therefore, it is advisable to use 3D images for high-quality visualization of theoretical material. For example, in the course “Mathematical Analysis” in the study of “Applied applications of integrals”, “Area of definition of the function of several variables” AR capabilities allow students to qualitatively visualize bodies defined by the condition of the problem, better understand theoretical material, process

of geometric constructions. This significantly improves the process of comprehension and memorization of educational material the learning includes a game element with dynamic elements of animation.

Today, a popular AR application among future teachers of physics, mathematics, computer science studying the course “Mathematical Analysis” is the mathematical package GeoGebra. A number of examples that can be used in teaching mathematics are offered by the developers of the dynamic mathematics system GeoGebra on the YouTube page (youtube.com/user/GeoGebraChannel/featured) and [7]. With GeoGebra 3D Graphing Calculator from AR, you can build bodies and surfaces of different geometries, create cross-sectional designs, complement physical reality with new objects, explore augmented reality objects in the middle of the body, save and share results.

Other applications AR MATH, Math Worlds AR, Math-O-Matic AR, Math Jumps: Math Games, Math Wiki - Learn Math are used less often.

Consider examples of the use of AR Geogebra to solve problems of the course “Mathematical Analysis” on the topics “Application of the integral” (functions of one and more variables) and “Area of definition of the function of several variables”. Geogebra Mathematical Package can also be used to calculate geometric quantities (areas, volumes) and physical quantities (masses, coordinates of the center of mass, work, pressure forces). However, we focus on the ability of the AR Geogebra tool to complement physical reality with digital model-objects that are specified in the task condition. Because in solving such problems the greatest difficulties arise when performing geometric constructions and visualization of mathematical objects. Qualitatively performed geometric constructions are one of the conditions for the correct solution of the problem.

Example 1. The calculation of the volume of the body formed by rotating around the abscissa of the figure bounded by lines: $y = 2x + 1$, $x = 1$, $x = 0$, $y = 0$.

Comment. To create a mathematical model in an augmented reality application, you must first create a model in 3D graphics, and only then use the “AR” button to project into the real world.

To place an object in the real world, you need to select a location, point it at the camera, and tap the screen while calling your tablet. After that the figure will be fixed on the chosen place. We use the touch screen to change the size and color of the object.

The application has the ability to build the correct flat shapes and bodies with built-in tools. But you can not build most of the bodies that are determined by the conditions of the problems on the topic “Applied applications of integrals” with the help of built-in tools of the mathematical package. Therefore, in the command line we prescribe analytical expressions that determine the geometry of the body. After constructing the body in 3D Graphics, click on the “AR” button. The next you need to use the camera to select the location in physical reality where you plan to move the object. For example, on the floor or table. By tapping the screen, the body will be transferred to the real world, where it can be explored. The camera of the device will serve as our eyes. Dipping the camera (device) into the virtual body, we will see it from the inside, we can go around it, and the application allows you to change the size, color. With augmented reality from GeoGebra, you can clearly see that we are surrounded by mathematical objects everywhere, explore them, walk around them, look or go inside the figure.

In figure 1 you can see the visualization of the mathematical object given by the condition of the problem. The volume of the body of rotation can be calculated using the tool “Integral”.

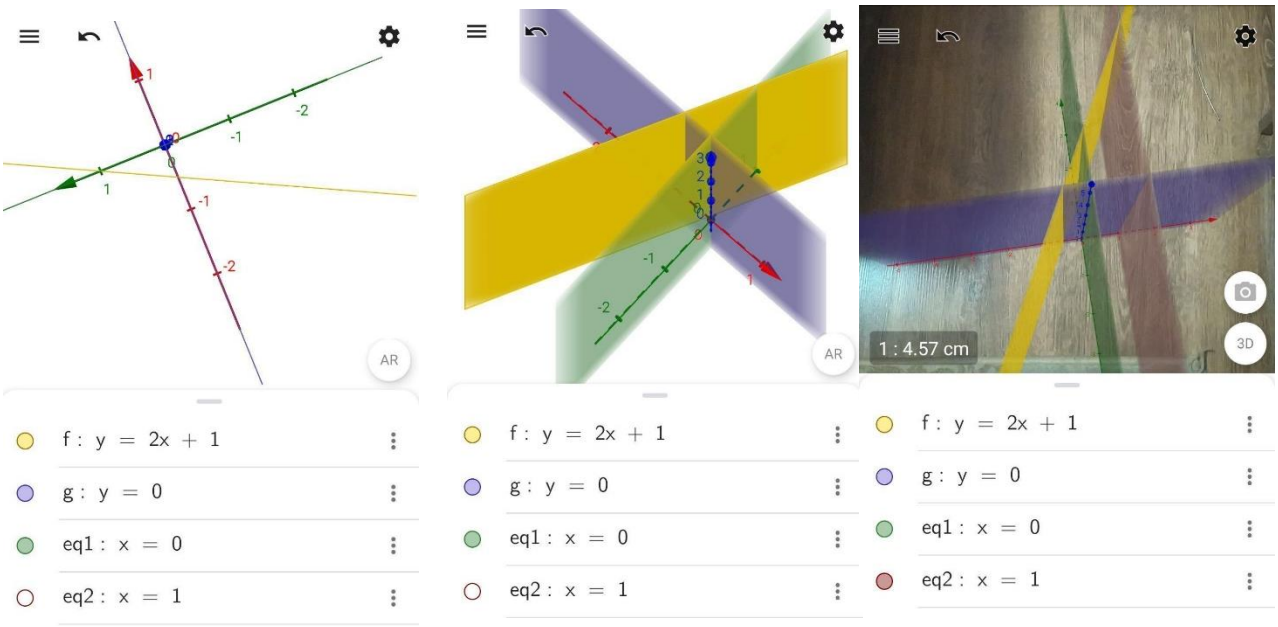

Figure 1. Visualization mathematical object in GeoGebra.

Example 2. The calculation of the area of the part of the paraboloid $x= 1-y^2-z^2$ that is cut off by the cylinder: $y^2+z^2 = 1$.

Comment. Follow the algorithm shown in example 1. We will build on 3D canvas. In the command line we write analytical expressions, which determine the geometric bodies given by the condition. So, we will immediately see the body defined by the condition of the problem. Choose AR mode and get a mathematical object that complements reality. In figure 2 you can see the visualization of the mathematical object given by the condition of the problem.

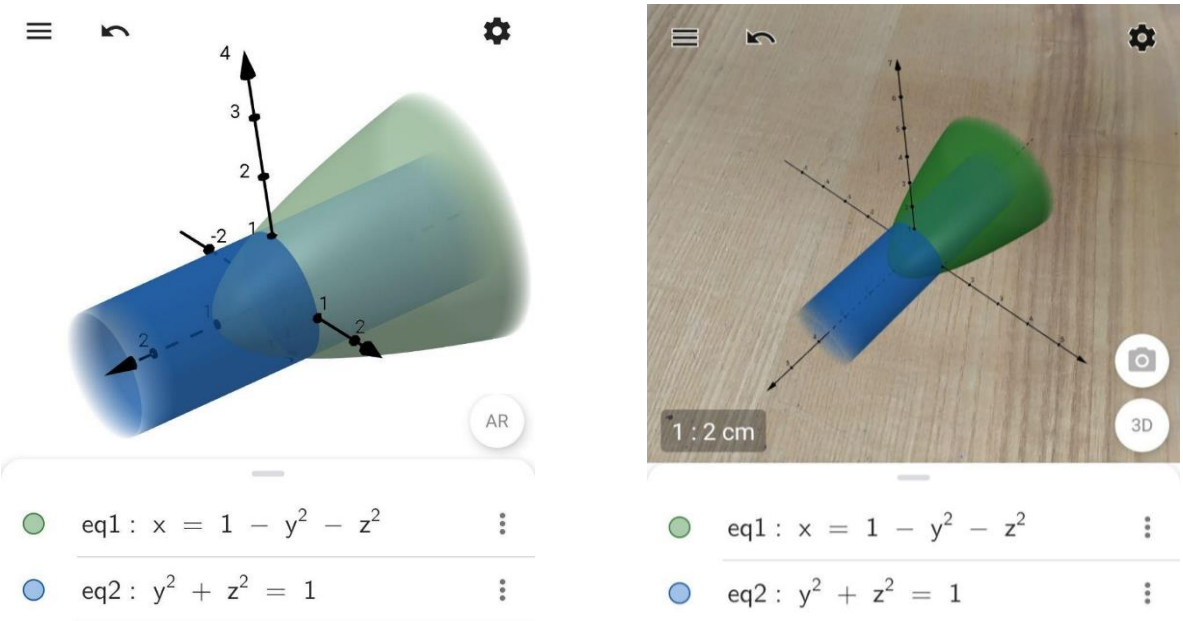

Figure 2. GeoGebra Augmented Reality.

Example 3. Find the domain of the function u=$Ln(4-x^2-y^2-z^2)$.

Comment. Follow the algorithm shown in example 1. But first, analytically determine the scope of the definition of the function: $x^2+y^{2+}+z2=4$. This will be the set of inner points of the sphere of radius 4. Write the last expression in the command line and see the body defined by the condition of the problem. We choose the AR mode and get a mathematical object that complements the physical reality (figure 3).

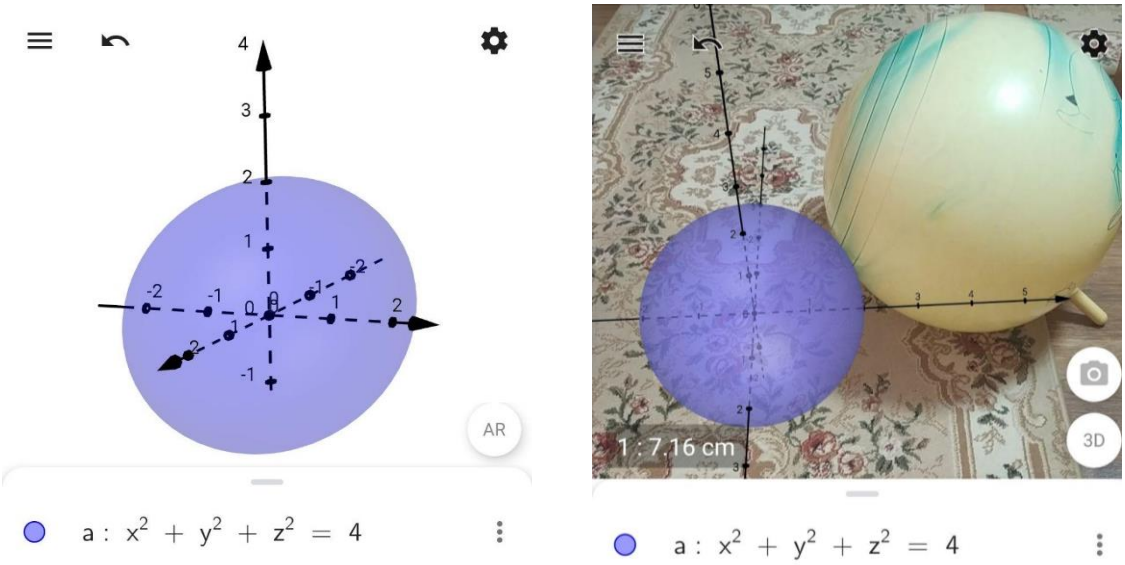

Figure 3. GeoGebra Augmented Reality.

During 2020-2021 and 2021-2022 a. y. students (120 people) of educational programs Secondary education (Physics), Secondary education (Mathematics), Secondary education (Mathematics. Informatics) and Secondary education (Informatics) during the individual homework of the course "Mathematical analysis" on "Geometric and mechanical applications of integrals "(function of one and several variables) and "Area of definition of the function of several variables " it was proposed to make a dynamic model of the body defined by the condition of the problem using various digital AR services. The distribution of preferences of digital resources and services for the creation of mathematical models in augmented reality among higher education seekers is presented in figure 4.

The clear advantage of Geogebra is its availability and students have the appropriate skills to use this package.

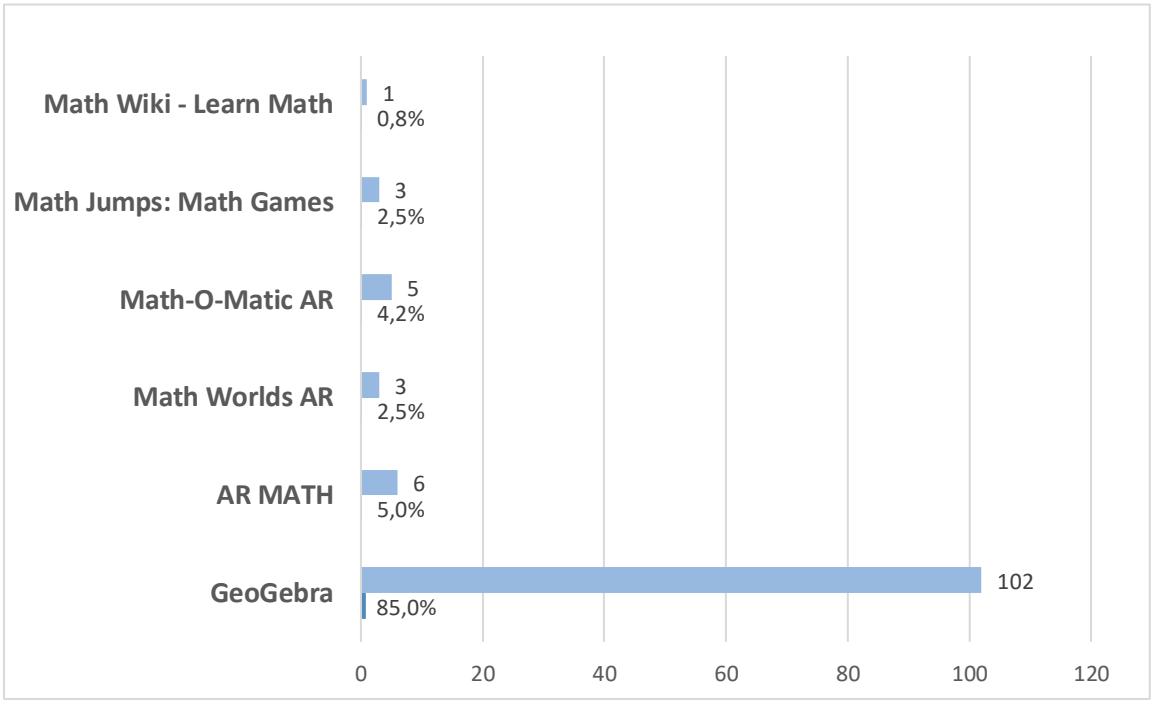

Figure 4. Distribution of preferences of digital resources among students.

In particular, a survey was conducted for students from which we can conclude that 87% of higher education students agree that AR technology has a positive impact on the teaching process, 82% believe that AR technology improves educational outcomes, 78% believe that AR technology helps to understand the presented material better, 84% said that AR technology increases motivation to learn (fig. 5).

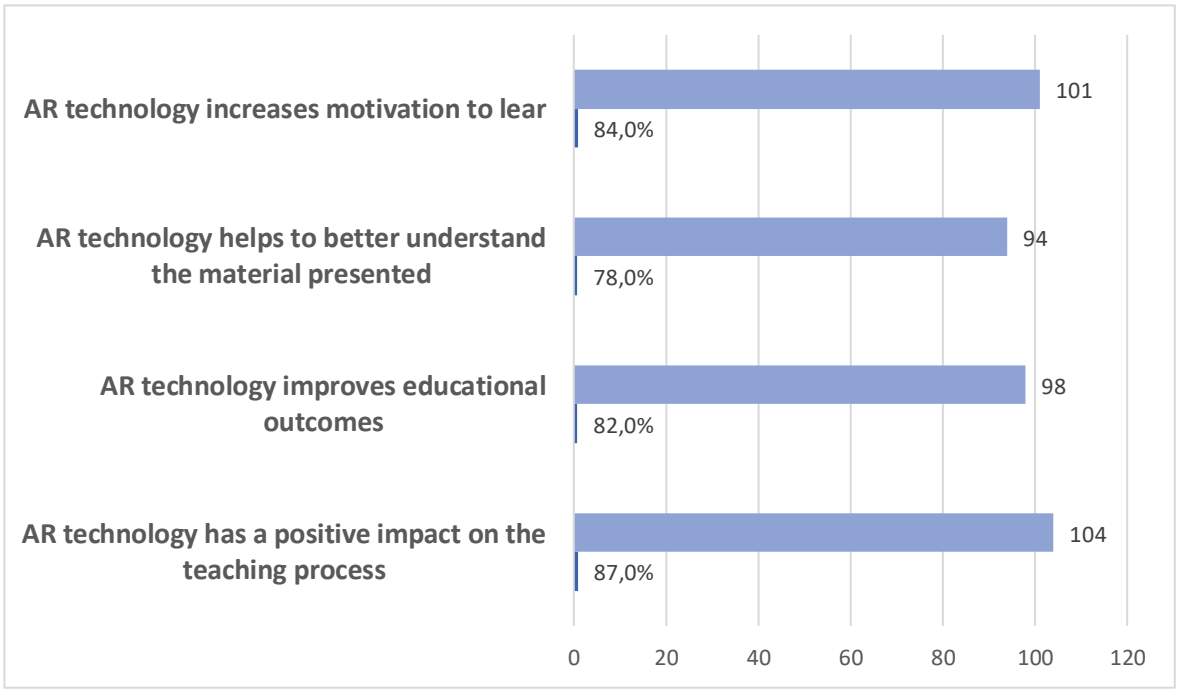

Figure 5. The results of the survey of students

3. Conclusions and prospects of further research

The usage of augmented reality technologies promotes visualization in the educational process, the formation of cognitive interest and motivation of students. It has significant methodological potential, which should be used in the study of various mathematical disciplines, in particular in the study of mathematical analysis. The use of such technologies can probably have a great effect, but the use of constant use in the standard practical training of 90 minutes leads to a significant violation of the program. However, the use of augmented reality technologies is the most adequate as an additional means of visualizing mathematical objects in solving problems.

The results of student surveys and trainings confirm the positive impact of AR technologies on the learning process and the quality of learning outcomes. A significant part of higher education students note that AR technology helps to better understand theoretical material and enhances the visual component.

4. References

[1]. A. Ashley-Welbeck & D. Vlachopoulos, "Teachers' Perceptions on Using Augmented Reality for Language Learning in Primary Years Programme (PYP) Education", *International Journal of Emerging Technologies in Learning (iJET)*, no. 15(12), p.p. 116-135, 2020. [Online]. Available: https://www.learntechlib.org/p/217568/. [Accessed on: March 19, 2022].

[2]. A. Frolli, A. Bosco, A. Lombardi, F. Di Carmine, S. Marzo, A. Rega, Angelo & M. Ricci, "Asperger's and virtual reality", *Proceedings of the First Workshop on Technology Enhanced Learning Environments for Blended Education* (teleXbe2021), January 21-22, 2021, Foggia, Italy. [Online]. Available: http://ceur-ws.org/Vol-2817/paper10.pdf [Accessed on: February 17, 2022].

[3]. A. Mohamed Yousef Fahmy "The applied AR technology allows pupils to handle 2D and 3D rounded corners and simple closed curves to find the most effective solutions for some learning challenges". *Journal of Computer Assisted Learning*, no.37(4), p.p. 966-977, 2021. [Online]. Available: https://doi.org/10.1111/jcal.12536. [Accessed on: February 20, 2022].

[4]. Abdullah M. Baabdullah, Abdulellah A. Alsulaimani, Alhasan Allamnakhrah, Ali Abdallah Alalwan, Yogesh K. Dwivedi & Nripendra P. Rana, "Usage of augmented reality (AR) and development of e-learning outcomes: An empirical evaluation of students' e-learning experience", *Computers & Education*, no. 177, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/abs/pii/S0360131521002608. [Accessed on: December 28, 2021].

[5]. B. Ancochea, M.-I. Cárdenas, "Exploring real world environments using potential of Geogebra AR, in: M. Ludwig, S. Jablonski, A. Caldeira, A. Moura (Eds.)", *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA.* June 2020, pp. 41–46. [Online]. Available: https://www.wtm-verlag. de/DOI-Deposit/978-3-95987-144-0/978-3-95987-144-0-Book.pdf. doi:https://doi.org/ 10.37626/GA9783959871440.0.05.

[6]. F. M. Dinis, A. S. Guimaraes, B. R.Carvalho & J. P. Martins, "Development of virtual reality game-based interfaces for civil engineering education", *IEEE Global Engineering Education Conference (EDUCON)*, p.p. 1195-1202. 2017. [Online]. Available: https://www.semanticscholar.org/paper/Development-of-virtual-reality-game-based-for-civil-Dinis-Guimaraes/a1432577fd5a5a55b78186b1a44290dcda029fbb [Accessed on: January 16, 2022].

[7]. GeoGebra - the world's favorite, free math tools used by over 100 million students and teachers, 2021. [Online]. Available: https://www.geogebra.org/.[Accessed on: December 20, 2021].

[8]. I. V. Kravets, L. Ya. Midak & O. V. Kuzyshyn, "Augmented Reality technology as a means to improve the efficiency of the study of chemical disciplines", All-Ukrainian scientific-practical conference with international participation "Modern information technologies and innovative teaching methods: experience, trends, prospects", November 9-10, 2017, Ternopil, Ukraine. p.p. 151-154.

[9]. I. Y. Melnyk, H. D. Nefedova & N. M. Zadyrey, "Augmented and virtual reality as a resource for students 'educational activities Information technologies and computer modeling", *International scientific-practical conference*, p.p. 61-64. 2018.

[10]. K. K. Bhagat, F. Y. Yang, C. H. Cheng, *et al.*, "Tracking the process and motivation of math learning with augmented reality", *Education Tech Research Dev*, no.69**,** p.p. 3153-3178. 2021. [Online]. Available: https://doi.org/10.1007/s11423-021-10066-9 [Accessed on: February 10, 2022].
[11]. M. Akçayır & G. Akçayır, "Advantages and challenges associated with augmented reality for education: A systematic review of the literature", *Educational Research Review*, no. 20, p.p. 1-11, 2016. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1747938X16300616 [Accessed on: January 13, 2022].
[12]. M. Huttar Carol & B. S. Karlynn, "Virtual Reality and Computer Simulation in Social Work Education: A Systematic Review", *Journal of Social Work Education*, no. 56 (1). p.p. 131-141, 2019. [Online]. Available: https://www.researchgate.net/publication/335487719_Virtual_Reality_and_Computer_Simulation_in_Social_Work_Education_A_Systematic_Review. [Accessed on: December 23, 2021].
[13]. M. Monfared, V. K. Shukla, S. Dutta & A. Chaubey, "Reshaping Education Through Augmented Reality and Virtual Reality. In: Tavares J.M.R.S., Dutta P., Dutta S., Samanta D. (eds)" Cyber Intelligence and Information Retrieval. Lecture Notes in Networks and Systems, no. 291. p.p. 619-629. 2022. [Online]. Available: https://link.springer.com/chapter/10.1007%2F978-981-16-4284-5_55 [Accessed on: January 05, 2022].
[14]. N. F. Murodillayevich, U. G. Eshpulatovich & J. O. Pardaboyevich, "Integration of virtual reality and 3D modeling use of environments in education", *2019 International Conference on Information Science and Communications Technologies (ICISCT)*, pp. 1-6. 2019. [Online]. Available: https://www.researchgate.net/publication/339558505_Integration_of_virtual_reality_and_3D_modeling_use_of_environments_in_education [Accessed on: January 15, 2022].
[15]. N. V. Osypova & V. I. Tatochenko, "Improving the learning environment for future mathematics teachers with the use application of the dynamic mathematics system GeoGebra AR" *Proceedings of the 4th International Workshop on Augmented Reality in Education (AREdu 2021).* CEUR Workshop Proceedings, May 11, 2021, Kryvyi Rih, Ukraine. no. 2898, p.p. 178-196. [Online]. Available: http://ceur-ws.org/Vol-2898/paper10.pdf [Accessed on: February 19, 2022].
[16]. S. Gargrish, D. P. Kaur, A. Mantri, G. Singh, B. Sharma, "Measuring effectiveness of augmented reality-based geometry learning assistant on memory retention abilities of the students in 3D geometry" , *Computer Applications in Engineering Education*, no. 29 (6), p.p. 1811-1824. 2021. [Online]. Available: https://doi.org/10.1002/cae.22424.
[17]. S. Lutvunova, O. Burov & S. Semerikov, "Conceptual approaches to the use of associated reality in the educational process". *Modern information technologies and innovative teaching methods in training: methodology, theory, experience, problems*, no.55, p.p.46-62. 2021. [Online]. Available: https://doi.org/10.31652/2412-1142-2020-55-46-62. [Accessed on: March 17, 2022].
[18]. S. S. Elesun & A. V. Feshchenko, "Virtual Reality in Education: Doubts and Hopes", *Humanitarian Informatics*. no. 10, p.p. 109-114, 2016.
[19]. T. A. Vakaliuk & S. I. Pochtoviuk, "Analysis of tools for the development of augmented reality technologies", *Proceedings of the 4th International Workshop on Augmented Reality in Education (AREdu 2021)*, CEUR Workshop Proceedings, May 11, 2021, Kryvyi Rih, Ukraine. no. 2898, p.p. 119-130. [Online]. Available: http://ceur-ws.org/Vol-2898/paper06.pdf [Accessed on: February 03, 2022].
[20]. T. Trust, N. Woodruff, M. Checrallah, & J. Whalen, "Educators' Interests, Prior Knowledge and Questions Regarding Augmented Reality, Virtual Reality and 3D Printing and Modeling", *TechTrends*, no. 65 (4), p.p. 548-561, 2021.
[21]. V. V. Babkin, V. V. Sharavara, V. V. Sharavara, V. V. Bilous, A. V. Voznyak & S. Ya. Kharchenko, "Using augmented reality in university education for future IT specialists: educational process and student research work", *Proceedings of the 4th International Workshop on Augmented Reality in Education (AREdu 2021).* CEUR Workshop Proceedings, May 11, 2021, Kryvyi Rih, Ukraine. no. 2898, p.p. 255-268. [Online]. Available: http://ceur-ws.org/Vol-2898/paper14.pdf. [Accessed on: January 10, 2022].
[22]. V. Volynets, "Use of virtual reality technologies in education", *Continuing Professional Education: Theory and Practice*, no. 2, p.p. 40-47, 2021. [Online]. Available: https://doi.org/10.28925/1609-8595.2021.2.5 [Accessed on: February 22, 2022].

[23]. V. Ye. Klumnyuk, “Virtual reality in the educational process”, *Collection of scientific works of Kharkiv National University of the Air Force*. no. 2(56), p.p. 207-212. 2018. [Online]. Available: https://www.researchgate.net/publication/325873912_Virtualna_realnist_v_osvitnomu_procesi. [Accessed on: March 17, 2022].