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ВІЗУАЛІЗАЦІЯ НАВЧАЛЬНОГО МАТЕРІАЛУ В ПРОЦЕСІ ПІДГОТОВКИ МАЙБУТНІХ УЧИТЕЛІВ МАТЕМАТИКИ ЗАСОБАМИ СЕРЕДОВИЩА GEOGEBRA

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USING GEOGEBRA ENVIRONMENT TO VISUALIZE EDUCATIONAL MATERIAL IN THE PROCESS OF TRAINING PRE-SERVICE MATHEMATICS TEACHERS

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АНОТАЦІЯ

Формулювання проблеми. Навчання майбутніх вчителів математики потребує високий рівень візуалізації навчального матеріалу. Інтеграція динамічного середовища GeoGebra у навчальний процес може допомогти покращити навички та знання студентів, а також підвищити рівень викладання для досягнення бажаних цілей навчання.

Матеріали і методи. У ході роботи використовувались наступні методи: теоретичні (аналіз науково-методичної літератури для дослідження стану проблеми застосування засобів візуалізації методами комп'ютерної математики); емпіричні (спостереження та систематизація інструментів GeoGebra на лекційних і практичних заняттях дисциплін «Лінійна алгебра» та «Диференціальна геометрія і топологія»). Показано, як за допомогою GeoGebra можна будувати та досліджувати просторові та плоскі криві; виконувати дії над матрицями.

Результати. Розглянуто особливості використання авторських аплетів та інших розробок GeoGebra під час викладання дисциплін диференціальна геометрія та лінійна алгебра у процесі підготовки майбутніх вчителів математики, зокрема описано можливості застосування даного інструмента для дослідження властивостей просторових кривих і формування практичних вмінь та навичок виконання операцій над матрицями, знаходження обернених матриць. Висвітлено переваги та недоліки використання середовища GeoGebra в освітньому процесі з лінійної алгебри та диференціальної геометрії.

Висновки. В сучасному світі впровадження інформаційних технологій в освітній процес є необхідною компонентою успішного засвоєння навчальних дисциплін з математики. Однією з потужних систем комп'ютерної математики для динамічної візуалізації, розрахунків під час розв'язування задач, обробки даних та науково-дослідницької роботи є середовище GeoGebra. В роботі на прикладі окремих математичних дисциплін показано переваги використання даної системи в освітньому процесі.

КЛЮЧОВІ СЛОВА: інформаційні технології; GeoGebra; динамічна математика; візуалізація; диференціальна геометрія; лінійна алгебра.

ABSTRACT

Formulation of the problem. The training of pre-service mathematics teachers requires a high level of visualization of educational material. Integrating a dynamic learning environment GeoGebra can help improve students' skills and knowledge, as well as improve the level of teaching to achieve desired learning goals.

Materials and methods The following methods were used in the course of work: theoretical (analysis of scientific and methodological literature to study the state of the problem of using visualization tools by computer mathematics); empirical (observation and systematization of GeoGebra tools in lectures and practical classes of the disciplines "Linear Algebra" and "Differential Geometry and Topology"). It illustrated how one can build and explore space and plane curves with GeoGebra as well as manipulate matrices.

Results. This work studies peculiarity of using custom applets and other GeoGebra activities in the process of teaching differential geometry and linear algebra as a part of the pre-service mathematics teachers' training curriculum. In particular, it describes the affordances of this tool for studying the space curves and forming matrices operations, and finding inverse matrices skills. The advantages and disadvantages of using the GeoGebra environment in the process of teaching linear algebra and differential geometry are highlighted.

Conclusions. In today's world, the introduction of information technology in the educational process is a necessary component of the successful mastering of mathematics. GeoGebra is one of the powerful systems of computer mathematics used for dynamic visualization, problem-solving calculations, data processing, and research. This work uses some of the mathematical disciplines to illustrate the benefits of using GeoGebra in the educational process.

KEYWORDS: information technologies; GeoGebra; dynamic mathematics; visualization; differential geometry; linear algebra.

INTRODUCTION

Formulation of the problem. The constant development of information technology contributes to the expansion of opportunities for receiving, storing, processing, and transmitting the information. Given the need to improve the system of higher education, the use of modern information technologies (IT) is a powerful factor in improving the efficiency of education and the training of qualified professionals. In particular, during the training of pre-service teachers of mathematics, the use of IT opens new opportunities to not only increase the quality of learning material and the formation of practical skills but also to develop general and professional competencies and increase motivation to learn. Technologies of visualization of educational material, in particular, the dynamic mathematical environment GeoGebra with its wide range of functionalities and uses in education hold a special place among modern learning technologies.

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Analysis of current research. Several findings by national and foreign scientists are devoted to the importance of introducing digital communication technologies into the process of education. These are works by Aliexsieiev O. M., Biliay Yu. P., Biloshapka N. M., Bykov V. Yu., Cherniavskiy B. V., Drushlyak M. H., Fedorenko O. H., Haletskiy S. M., Herasymenko I. V., Karandina S. I., Kovalchuk M. O., Kruhlyk V. S., Letsiuk I. Z., Lytvynova S. H., Mashbyts E. I., Medvedieva M. O., Mitieniev Yu. A., Mykytenko A. P., Myroshnichenko Yu. B., Nechypurenko P. P., Petrenko S. I., Rashevskaya N. V., Semenikhina O. V., Shrol T. S., Shylyna H. A., Slovák K. I., Snihur O. M., Spirin O. M., Spivakovskiy O. V., Tkachuk H. V., Tkachuk V. V., Tryus Yu. V., Tsys O. O., Zhuk Yu. O., and others.

Various aspects of the introduction of visualization in the educational process are widely considered in their research by domestic and foreign scientists, in particular Argul A., Bezuhlyi D., Bilousova L. I., Briantseva V. H., Brucker B., Drushlyak M. G., Gerjets P., Hurevych R. S., Imber V. I., Semenikhina O. V., Shaiter K., Yamet I., and others.

Bezuhlyi D. notes that the visualization of information is a very important component of the didactic process (Bezuhlyi, 2014). Cognitive-visual approaches allow to not only present a large amount of information in a concise form but also promote more productive learning activities, enhance cognitive interest and development of associative thinking, transfer methods of information processing to various objects and spheres of society.

Brucker B., Scheiter K., and Gerjets P. who compared the effectiveness of dynamic visualization with static visualization note that students who were offered teaching material in the form of dynamic visualization understood it better than those who were presented with static visualization (Brucker et al., 2014).

Argul A. and Yamet I. explain the expediency of using visualization tools in the educational process and the need to combine the representation of dynamic visualization with static (Arguel & Jamet, 2009).

Several findings are devoted to the peculiarities of introducing GeoGebra into the process of education: research by Drushlyak M. G., Hrybyuk O. O., Ishchenko H. V., Lukashova T. D., Polishchuk T. V., Semenikhina O. V., Skaskiv L. V., Voznosymenko D. A., Yaremenko Yu. V., Yunchyk V. L. and others. Polishchuk T. V., who studied GeoGebra's affordances for mathematical analysis problems, said that the use of the environment allows for optimization of the learning process, conducting individual work, improving learning, enhancing cognitive activity, and promoting the formation of algorithmic thinking in pre-service teachers of mathematics, creating an opportunity to demonstrate the results of their educational activities, and most importantly, increasing students' interest in basic science by allowing visualization of the use of mathematical apparatus to solve applied problems (Polishchuk, 2019).

Drushlyak M. G., Lukashova T. D., Skaskiv L. V. confirmed the effectiveness of solving graph theory problems using GeoGebra, noting that the use of GeoGebra allows to not only to solve typical problems of the course but also to connect each problem with real-life situations through visualization (Drushlyak et al., 2019).

A significant amount of research is devoted to the use of GeoGebra in the teaching of algebra and geometry in general secondary education, as well as the teaching of certain mathematical disciplines in higher education, but the use of GeoGebra in teaching mathematics in the training of pre-service mathematics teachers is not sufficiently covered in the scientific literature. Therefore, it needs to be further researched.

Purpose of the article. Explore the affordances and reasons for using GeoGebra in the process of teaching about the elements of algebra and geometry in higher education institutions.

METHODS OF THE RESEARCH

The following methods were used in the research process: theoretical (analysis of scientific and methodological literature to investigate the problem of visualization of educational material in mathematics, comparison and synthesis of theoretical positions), empirical (observation and implementation of tools of dynamic system GeoGebra in teaching of certain mathematical disciplines in high school, generalization of pedagogical experience).

RESEARCH RESULTS AND DISCUSSION

During the process of training pre-service teachers of mathematics, modern teaching means are used in several ways: to visualize educational material; to assist with self-study of educational material by students; to organize the evaluation of students' learning outcomes; for teacher-student interaction in cases where full-time learning is impossible or difficult; as simulators for practicing practical skills.

There is an increasing need to use modern teaching means in teaching mathematics to students of pedagogical universities. This is due to several factors:

1. The need to motivate students to study mathematical disciplines.
2. Reduction of classroom hours for the study of the discipline, while the program learning outcomes remain unchanged and there is a need to intensify the learning process.
3. Low level of the students' basic knowledge.
4. The need to visualize educational material.

One of the modern and multifunctional means of visualization of educational material in the teaching of fundamental mathematical disciplines in pedagogical universities is the free of charge dynamic mathematical environment GeoGebra. GeoGebra can be used to understand certain mathematical concepts, to help solve problems, and check their solutions.

As noted by scientist Semenikhina O. V. there are several ways of using the cloud service GeoGebra in teaching pre-service teachers of natural sciences and mathematics to introduce visualized mathematical content: as an environment to organize both analytical and empirical solutions to the problems in the process of determining individual characteristics of mathematical objects and as an environment for at-home computer experiments (Semenikhina et al., 2019).

The GeoGebra environment can be successfully used to visualize educational material in the process of studying geometry and topology. Commonly, when studying differential geometry, students use known formulas and equations, successfully calculate the values of its differential-geometric invariants for a given curve (plane), find the equations of related

lines and planes, but do not make a connection to the shape of the curve (or plane). Such a formal approach to the training of pre-service mathematics teachers may further negatively affect their professional activities. The GeoGebra environment, due to its extensive functionality, can help solve this problem.

During the lecture on the content module "Space curves", GeoGebra can be used to set space curves and points that belong to them. Space curves in differential geometry are often set parametrically:

$$x = x(t), y = y(t), z = z(t)$$

or as one vector equation

$$\vec{r}(t) = (x(t), y(t), z(t))$$

This method is best used for setting up and recognizing GeoGebra. In the application GeoGebra 3D this is done by the command *Curve* (<Expression>, <Expression>, <Expression>, <Parameter Variable>, <Start Value>, <End Value>). On the example of a regular (cylindrical) helix, consider the image of a curve, the equation of which includes the letter coefficients (*a* and *b*). To do this, the command *Curve* (*a*cos(t)*, *a*sin(t)*, *b*t*, *t*, -10, 10) is used. After pressing the Enter key on the keyboard and in the GeoGebra 3D system, sliders for the coefficients *a* and *b* will automatically appear (Fig. 1.). As they are changed, one gets a real-time illustration of how changing one numerical coefficient at a fixed other affects the curve image (as changing the parameter *a* changes the radius of the cylinder containing the helix, while changing the parameter *b* – changes its pitch). One can also investigate some exceptional cases (for example, when one of the coefficients is zero). In addition, by clicking on the *Play* icon next to each parameter and adjusting the playback speed, one can observe the dynamic movement of the formation of the helix while changing both coefficients. Furthermore, the increase in the rate of change of the parameter *t* will increase the number of turns of the helix.

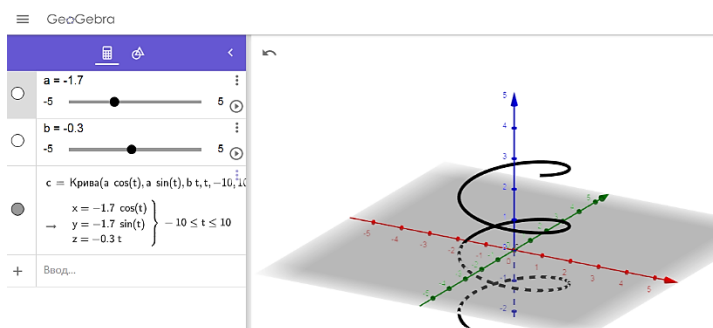


Fig. 1. Image of a helix

It is also possible to build closed space curves in the system that contain a self-intersection point, such as Viviani's curve (Fig. 2.). To do this, GeoGebra 3D uses the command *Curve* (*a/2 (1+ cos (t))*, *a/2 sin(t)*, *a*sin (t/2)*, *t*, -2pi, 2pi)

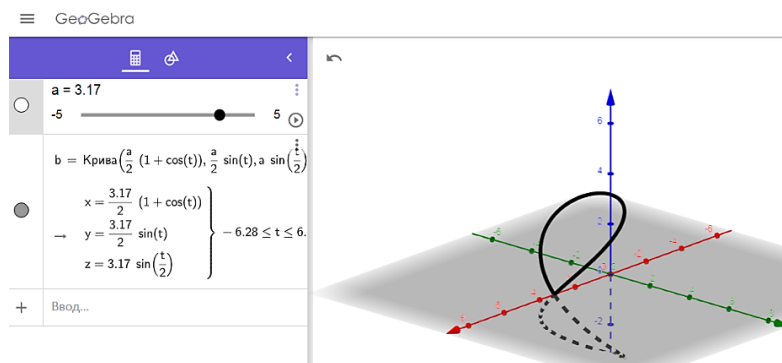


Fig. 2. Image of the Viviani's curve

The image of space curves is not the only possible cause of difficulties for students. Many lines on the plane are also of great interest for research in applied aspects of mathematics and can be constructed using GeoGebra media (Drushliak & Semenikhina, 2021). They are cycloid, epitrochoid, Fermat's spiral, deltoid, etc. The study of such curves can be offered to students as extracurricular work to be performed in the form of individual research work using the GeoGebra system. The subject of research can be the history of their discovery, practical application, and finding further characteristics of these curves.

The GeoGebra system can be used to visualize the training material used during practical classes. Integrating innovative teaching methods using IT can be extremely effective for teaching Linear Algebra or Analytical Geometry and Linear Algebra, as it helps activate students' learning. (Makhometa et al., 2018). For example, a series of applets "Actions on matrices" allows students to practice multiplication of a matrix by a number, adding matrices, and multiplying matrices during a practical lesson in the discipline "Linear Algebra". Methodological features of using a series of developed applets during a practical lesson on linear algebra were studied.

At the stage of actualization of basic knowledge, with the help of, for example, a face-to-face interview, students are reminded of the rules of operations on matrices. After that, the teacher displays the first applet with an animation video of the rule of multiplication of matrices on the screen or multimedia projector (Fig. 3). No such animations to demonstrate other operations have not been developed, as other operations usually do not cause difficulties for students.

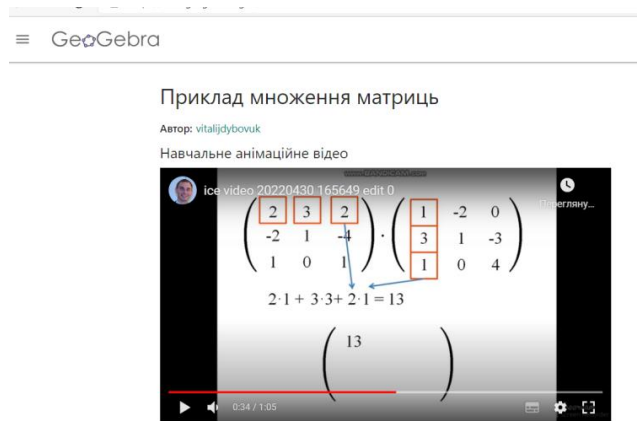


Fig. 3. The activity of GeoGebra with an animation video of the rule of multiplication

The formation of practical skills takes place with the help of applets that display actions on matrices with elements that are randomly selected by the program (-10 to 10), the refresh button, which sets new random elements of matrices, the button that displays the result (Fig. 4.). First, students solve the problems they see on the screen or the multimedia board, and then, by clicking on the result button, they check their answers. If the answer is incorrect, the students look for the mistake on their own. The teacher helps each student who has questions in the process.

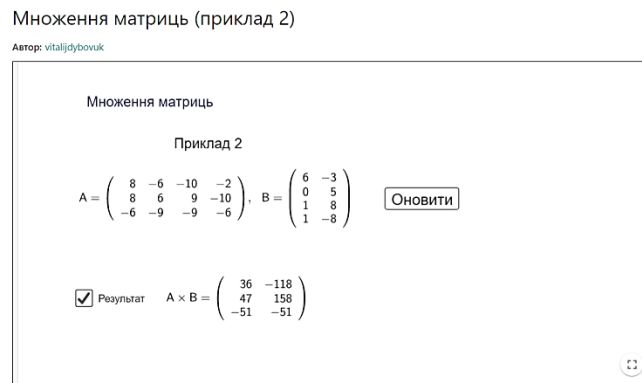


Fig. 4. The task in the GeoGebra software

Access to a series of applets is provided to students at the end of the lesson so that students can improve their skills and abilities to perform operations on matrices at home.

In the formation of practical skills and abilities to find inverse matrices the applet "Inverse matrix Gauss-Jordan" can be successfully used (Fig. 5.). This applet involves finding the inverse matrix to the matrix provided by employing elementary transformations. The user can choose the size of the matrix to which they find the inverted one and then fill in the appropriate cells of the matrix element. After clicking the arrows in the applet window, the steps of obtaining the inverse matrix to the specified one are visualized so that the user sees both the result and the operations performed on the rows.

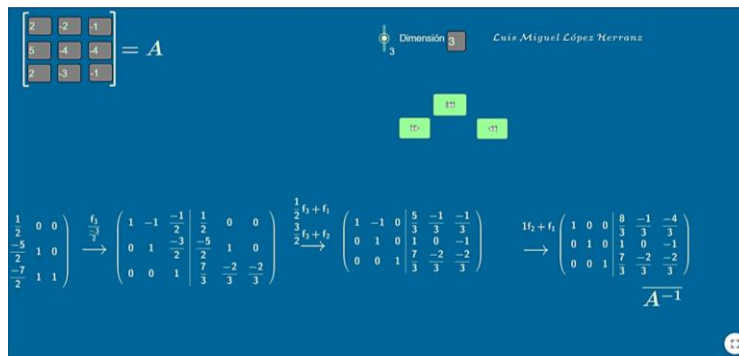


Fig. 5. Using the GeoGebra system to find inverse matrices
Source: <https://www.geogebra.org/m/jyZkAw>

Developing the practical skill of finding inverse matrices using this applet helps save time due to the lack of need to perform routine calculations, which, in turn, eliminates errors and incorrect answers.

GeoGebra environment can be used successfully to visualize educational material during teaching and other mathematical disciplines: mathematical analysis (Dereza & Ivanova, 2018), complex analysis (Polishchuk, 2021), elementary

mathematics (Kramarenko, 2019), graph theory (Drushlyak et al., 2019), analytical geometry (Drushlyak et al., 2020), mathematical foundations of computer science (Hrybiuk & Yunchyk, 2016), etc.

There are many benefits to using GeoGebra in teaching pre-service math teachers. In particular:

- allowing to visualize spatial models, stages of solving problems;
- promoting the individualization and differentiation of learning;
- contributing to the improvement of professional skills of pre-service teachers of mathematics by forming in pre-service teachers of mathematics information culture and skills in the use of computer technology and mathematical software products;
- intensifying educational and cognitive activities;
- integrating GeoGebra applets into electronic manuals and textbooks, or electronic educational and methodological complexes.

However, using the GeoGebra system has its disadvantages:

- the need for tools to work with the Geogebra service, as well as, possibly, an Internet connection;
- insufficient level of computer literacy and methodological preparation for practical work using GeoGebra;
- misuse of the system by the students, which can lead to poor learning of educational material.

CONCLUSIONS AND PERSPECTIVES FOR A FURTHER RESEARCH

The quality of the educational process significantly increases with the use of information technologies, in particular, the system of dynamic mathematics GeoGebra. This system becomes an effective tool in the visualization of educational material in the study of mathematical disciplines and helps improve the quality of knowledge of pre-service teachers of mathematics, intensification of educational and cognitive activities, and the formation of skills to acquire knowledge independently. As for the future research directions, they are the development of custom GeoGebra applets and compiling the methodology of teaching employing these tools in the process of training pre-service teachers of mathematics.

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