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# TWISTING RIGIDITY OF REINFORCED CONCRETE ELEMENTS OF I–BEAM SECTION WITH INCLINED CRACKS

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**Annotation.** The article shows that to determine the torsional stiffness of an element, one should first dissect the reinforcement in the place of an inclined crack. After cutting the reinforcement, the mutual displacement of the crack edges should be determined. This problem is the main and most difficult one in the general problem of determining the torsional stiffness of elements with both normal and inclined cracks. The article is devoted to solving just this most difficult part of the problem – determining the mutual displacement of the sides of an inclined crack of an I-beam element. According to the proposed technique, a real element with an oblique crack is replaced by an element with different stiffness in sections. Within the inclined crack, the element has a real slope equal to the slope of the inclined crack. In the area behind the tip of the inclined crack, it is assumed that the height of the element section changes from a height equal to the height of the zone above the crack to the full section height. Moreover, the change in height occurs according to the law of a straight line. This line is inclined at some angle to the horizontal.

**Key words:** torsion, inclined crack, torsional stiffness, movement in a crack, I-beam, Saint-Venant's principle.

**Introduction.** Redistribution of efforts between individual elements of complex statically indefinite systems depends on the ratio of their characteristics of rigidity [3, 5, 6, 8]. Flexural stiffness of reinforced concrete elements, taking into account the formation of cracks, nonlinear properties, have been studied quite widely. At the same time, much less attention has been paid to the issues of changing the torsional stiffness of cracked reinforced concrete elements. As a result, in the norms of Ukraine and many countries of the world, there are practically no methods for determining the stiffness and deformability of reinforced concrete elements with cracks.

It has long been believed that longitudinal reinforcement does not affect the torsional strength of a reinforced concrete element. A limited number of works have been devoted to the calculation of the stiffness and torsional strength of reinforced concrete elements with normal cracks. These are some works where elements of rectangular, box-shaped and hollow triangular sections were considered. However, a large class of reinforced concrete elements have a cross-section in the form of an I-beam, which leaves a significant imprint on their stress-strain state during torsion.

**Research analysis and problem statement.** In reinforced concrete elements, bending and torsional stiffnesses are significantly affected by various cracks. A fairly large number of works [1, 12] are devoted to the issues of changing bending stiffnesses. Considerably fewer works are devoted to the issues of determining the displacements during torsion of reinforced concrete elements [9-11]. In these and other works, the presence of spatial spiral cracks is assumed. However, such techniques are not suitable for calculating torsional displacements of elements with normal and oblique cracks that are formed from bending moments. At the same time, there is a large class of structures exposed to both bending and torsion moments, in the elements of which only normal and oblique cracks are formed. These are ribs of ribbed prefabricated and monolithic floors, crossbars, etc. [3]. The works [1, 2, 14] are devoted to the problem of determining the torsional stiffness of reinforced concrete elements with normal cracks. In these works, it is shown that such a problem should be divided into three stages: at the first stage, the longitudinal

reinforcement is conventionally cut and the mutual displacement of the edges of a normal crack is determined; at the second stage, the thrust forces in the longitudinal reinforcement are determined; the third stage is the determination of the torsional stiffness of the element with known thrust forces. The main and most difficult task is the first stage. This is since the use of the formulas of the theory of elasticity to determine displacements in this case is not possible since the torque is transmitted through a part of the section of the element. There are no works to determine the torsional stiffness of reinforced concrete beams with inclined cracks.
In the connection with the above, **this article aims** to develop a methodology for calculating displacements during torsion of an I-beam element with an inclined crack.

**The object of research** is the work during torsion of reinforced concrete I-beam elements with inclined cracks.

**Research methods** – methods of structural mechanics (when developing a method for determining the displacement of the sides of a normal crack); a computer program *Excel* using *Visual Basic* for analyzing the stress-strain state of the elements under consideration according to the developed method; numerical studies using the *Lira* program – to compare data with the results of calculations according to the developed method.

**Research results**. As mentioned above, the main and most difficult in the general problem of determining the stiffness of a reinforced concrete element with a normal or inclined crack is to determine the mutual displacement of the crack edges with already conditionally dissected longitudinal reinforcement. Consider an element of arbitrary section with an oblique crack, the right end of which is sealed, and a torque is applied to the left end (Fig. 1). The main task in this case is to determine the mutual displacement of points 4 and 5 in the crack under the action of the torque $\mathbf{M_t}$.

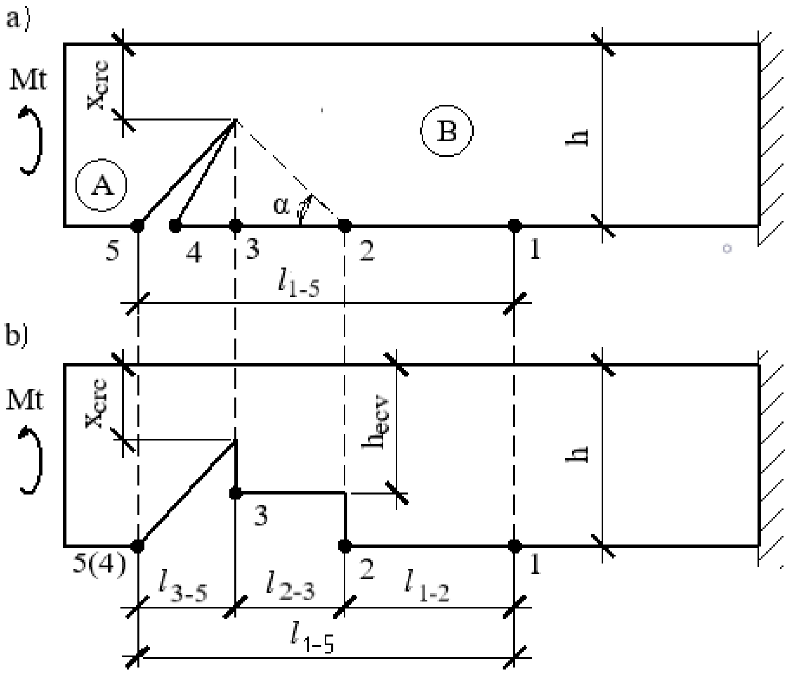


Figure: 1. Diagram of an element with an inclined crack (a) and its representation in the form of a stepwise change in stiffness (b)

The torque from block A to block B is transmitted through a part of the section of height $x_{crc}$. This is the main difficulty in determining the displacements in the element subject to torsion. The methods of the theory of elasticity for determining displacements in a swirling element assume that the torque is transmitted by tangential forces distributed over the entire end section [4]. In the case of a normal or oblique crack (Fig. 1), this moment is transmitted by tangential forces distributed only on a part of the end section of block B [1, 2].

The twist angle in the section 3–5 can be easily determined by the known methods of resistance of materials [13] as an element with a variable section height. The definition of the twist angle in section 1–3 in the case of an oblique crack does not differ at all from its determination in the case of a normal crack. In connection with the above, following [2], where it was shown that in the case of a normal crack, the element to the right of the crack can be calculated as a conditional element with a section, the height of which changes along a certain curve, we imagine that in our case the element height changes according to the law of a straight line lines from small value $x_{crc}$ to full height h. This line in Fig. 1 is shown by dashed lines with an angle of inclination to the horizontal α. Typical areas in Fig. 1 are designated by numbers 1 ... 5.

On a section $l_{3-5}$ long, the element has a real slope of the lower face. On a section $l_{2-3}$ long, the element has a conditional slope from the height $x_{crc}$ to the height *h*.

The desired angle of rotation between points 4 and 5 will be determined from the model:

$$\varphi_{4-5} = \varphi_{1-5} - \varphi_{1-4}. \tag{1}$$

In turn, the angle of rotation $\varphi_{1-4}$ will be equal to the sum of the angles of rotation:

$$\varphi_{1-4} = \varphi_{1-2} + \left(\varphi_{2-3} \approx \varphi_{2-4}\right). \tag{2}$$

Approximate equality in brackets of the model (2) is obvious from Fig. 1, a. It was also verified by calculations in Lira program using volumetric finite elements. Indeed, points 3 and 4 are the points of the unloaded end of the element, twisted by the tangential forces applied to the upper part of its section with the height $x_{crc}$.

On an inclined section 3-5, the angle of rotation will be determined using the known approach of resistance of materials [13] as for an element with a variable section height. Let's call this angle $\varphi_{3-5}$ and consider it known. The angle of rotation between points 1 and 2 $\varphi_{1-2}$ is also easily determined by the well-known formula for the strength of materials [13]:

$$\varphi_{1-2} = \frac{M_t l_{1-2}}{GJ_{tot}}, \tag{3}$$

here $G_{Jtot}$ – torsional stiffness of the full section (element with full height h).

Point 1 in Fig. 1 is randomly selected.

To determine the angle of rotation on a section with a length of $l_{2-3}$, we assume that this section can be replaced by an element of a conditionally constant section with a height $h_{ekv} = (x_{crc} + h)/2$. Then the element diagram with an inclined crack according to Fig. 1, a will be replaced by a scheme with a stepwise change in stiffness shown in Fig. 1, b. The angle of rotation in a section of length l2-3 will also be determined from the known expression for the resistance of materials:

$$\varphi_{2-3} = \frac{M_t l_{2-3}}{GJ_{ekv}}, \tag{4}$$

here $GJ_{ekv}$ – equivalent stiffness of a member of a conditionally constant section height $h_{ekv}$.

The length $l_{3-5}$ is, in fact, the projection of the inclined crack onto the longitudinal axis of the element, which is determined by well-known techniques [7, 12].

The length $l_{2-3}$ is easily determined from geometric constructions with the known value of $x_{crc}$ and the angle of inclination α.

It should be noted that if the value of the equivalent height $h_{ekv}$ is such that $h_{ekv} > h - h_{f1}$ (where $h_{f1}$ is the thickness of the upper flange of the I-beam), then the moment of inertia $J_{ekv}$ in expression (4) will already be determined as for an I-beam with a lower flange, the thickness of which is equal to $h - h_{ekv}$. If the height $h_{ekv}$ is such that $h_{ekv} \le h - h_{f1}$, then the moment of inertia $J_{ekv}$ in expression (4) will be determined as the moment of inertia of the T-element.

Calculations using the above method show that the angle *α* of the slope of the line of change in the design height of the section to the horizontal should be taken equal to 45 degrees. Refinement of this angle or acceptance of a line not in the form of a straight line, but the form of a certain curve, is the subject of further research. Here we only note the fact that the length of the $l_{2-3}$ section will be within the Saint-Venant's length. Indeed, according to the Saint-Venant's principle, the uniform distribution of tangential stresses (transmitted in Fig. 1 from left to right) will be at a distance from the place of application of tangential forces (in our case, on a section of height $x_{crc}$ at point 3) equal to the larger size of the cross section [2].

To check the calculation method, a rectangular beam with an inclined crack was modeled in the Lira program. The beam section width is 100 mm, its height is 200 mm. Other dimensions are shown in fig. 2, a. In addition, this beam was calculated as a bar with a variable cross-section (Fig, 2, b) according to the method proposed above. The deformation modulus in both cases is taken as E= 25000 MPa.

The angle of rotation at the level of point a was compared with respect to the embedment (point O in Fig. 2). This angle in the model from volumetric finite elements was calculated by the formula:

$$\varphi_{O-a} = \frac{x_b + x_a}{h}, \tag{5}$$

Here $x_b$, $x_a$ – horizontal displacement, respectively, points *a* and *b*. The angle of rotation of the rod scheme was determined by the well-known formula for the strength of materials. The difference in the angles of rotation for the rod scheme and the scheme of volumetric finite elements was 2.4%, which indicates a sufficiently high calculation accuracy according to the proposed engineering method.

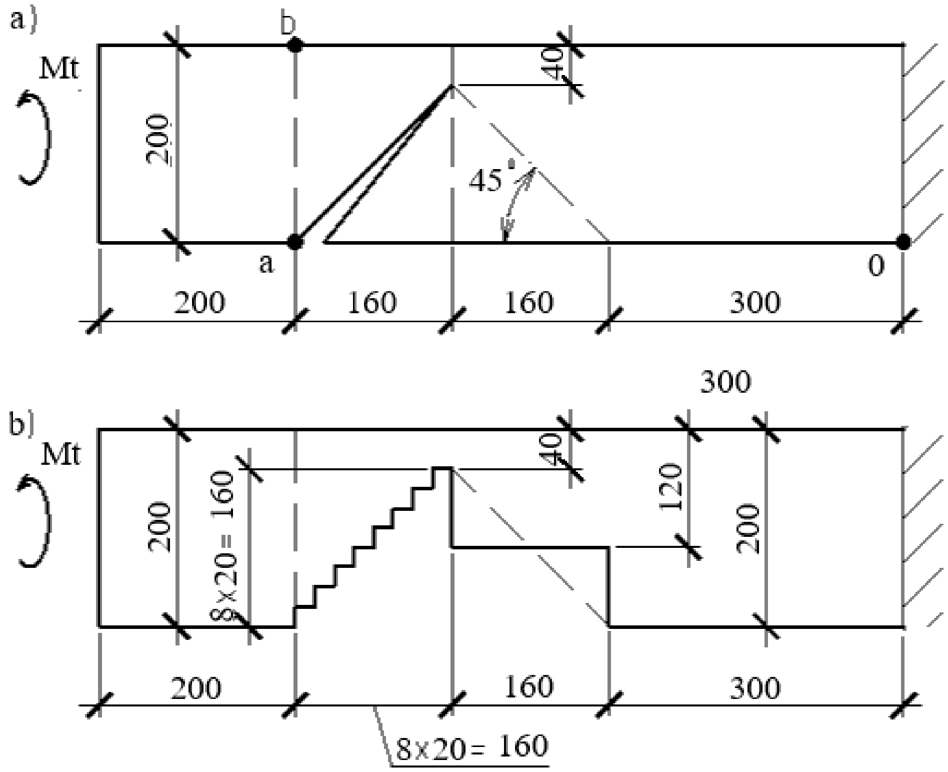


Figure: 2. Scheme for comparing the results:

a – element with an oblique crack, modeled in the Lyra software by volumetric finite elements; b – the same element, represented by rods with a step change in the section height

It should be noted that the calculation according to the proposed method differs from the calculation of an element with a normal crack only in that, in the case of an inclined crack, there is a section on the left side with a real inclination of the section (see Fig. 1 and 2). The rest of the task is identical. Using this technique, for the case of a normal crack, a comparison was made with calculations in the Lira program using volumetric finite elements. Table 1 below shows the data for such a comparison. An I-beam with a full section height *h = 220 mm* was considered. The width $b_{f1}$ of the upper flange and its thickness hf1 were varied; width and thickness of the bottom flange $b_{f2}$ and $h_{f2}$; height $h_3$ and thickness t of the wall. In addition, the crack height $h_{crc}$ and the distance between the cracks $l_{crc}$ were varied. Angle $\alpha$ in the design scheme according to Fig. 1 was taken equal to 45 ° (the angle was taken for a line outgoing from the top of a normal crack). The table shows the displacements from mutual rotation between the points of two adjacent normal cracks, calculated using the proposed method and in the Lira program using volumetric finite elements.

Table 1– Comparison of displacements determined by the developed technique and by the Lira program using volumetric finite elements

| № | $bf_1$ (м) | $hf_1$ (м) | $bf_2$ (м) | $hf_2$ (м) | $t$ (м) | $h_3$ (м) | $h_{crc}$ (м) | $l_{crc}$ (м) | $\Delta_{teor}$ (мм) | $\Delta_{Lira}$ (мм) | Error $\delta$ % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.110 | 0.30 | 80.65 | 87.81 | 8.15 |
| 2 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.123 | 0.30 | 84.37 | 91.80 | 8.09 |
| 3 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.174 | 0.30 | 102.37 | 106.64 | 4.00 |
| 4 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.142 | 0.30 | 90.48 | 97.69 | 7.38 |
| 5 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.045 | 0.30 | 65.72 | 70.00 | 6.11 |
| 6 | 0.30 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.058 | 0.30 | 68.22 | 74.73 | 8.72 |
| 7 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.110 | 0.30 | 105.88 | 111.22 | 4.80 |
| 8 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.123 | 0.30 | 112.09 | 117.69 | 4.76 |
| 9 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.174 | 0.30 | 143.00 | 143.56 | 0.39 |
| 10 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.142 | 0.30 | 122.42 | 127.58 | 4.04 |
| 11 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.045 | 0.30 | 81.56 | 83.78 | 2.66 |
| 12 | 0.20 | 0.03 | 0.09 | 0.05 | 0.03 | 0.15 | 0.058 | 0.30 | 85.55 | 90.55 | 5.52 |
| 13 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.110 | 0.30 | 43.92 | 49.02 | 10.41 |
| 14 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.123 | 0.30 | 45.08 | 50.28 | 10.35 |
| 15 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.174 | 0.30 | 50.48 | 54.21 | 6.88 |
| 16 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.142 | 0.30 | 46.99 | 52.01 | 9.66 |
| 17 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.045 | 0.30 | 39.12 | 43.53 | 10.14 |
| 18 | 0.30 | 0.04 | 0.09 | 0.05 | 0.03 | 0.14 | 0.058 | 0.30 | 39.95 | 45.31 | 11.84 |
| Average value % | | | | | | | | | | | 6.9 |
| coefficient of variation % | | | | | | | | | | | 0.45 |

These table data indicate the sufficient accuracy of the proposed engineering method. It should be noted that when performing calculations according to the proposed method, the torsional stiffness of the T-shaped elements was determined as the sum of the stiffness of the rectangles that make up the T-shaped. If the stiffness of the brands is determined by the exact method [4], then the coincidence of the results with the data calculated by the Lira program will be even greater.

**Conclusions and prospects of research**. A method for determining the mutual displacement of the banks of an inclined crack of an I-beam element is proposed. It is proposed to replace a real element with an inclined crack with an element with a step change in height and consideration in sections of equivalent rigidity. The line for changing the conditional height of the section from the top of the inclined crack is inclined at an angle to the horizontal, which is taken equal to 45°. The calculation technique allows determining the displacements in the crack without the use of cumbersome calculations. At the same time, it has sufficient accuracy for engineering calculations.

In the future, it is assumed to vary the angles of inclination to the horizontal of the line of change of the conditional height of the section, as well as the replacement of the straight line with a curve to refine the calculation results.

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# КРУТИЛЬНА ЖОРСТКІСТЬ ЗАЛІЗОБЕТОННИХ ЕЛЕМЕНТІВ ДВОТАВРОВОГО ПЕРЕРІЗУ З ПОХИЛИМИ ТРІЩИНАМИ

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**Анотація**. Для визначення жорсткості на елементі слід спочатку розрізати арматуру в місці похилої тріщини. Після розрізу арматури слід визначити взаємне зміщення берегів тріщини. Це завдання є основною і найскладнішою в загальному завданні визначення жорсткості на елементі як з нормальними, так і з похилими тріщинами.

Стаття присвячена вирішенню завдання – визначення взаємного переміщення берегів похилої тріщини елемента двотаврового перерізу. Згідно із запропонованою методикою реальний елемент з похилою тріщиною замінений елементом з різними твердостями по ділянках. В межах похилої тріщини елемент має реальний нахил, рівний нахилу похилої тріщини. На ділянці за вершиною похилої тріщини прийнята гіпотеза, що висота перерізу елемента змінюється від висоти, що дорівнює висоті зони над тріщиною, до повної висоти перерізу. При цьому зміна висоти відбувається за законом прямої лінії. Ця лінія нахилена під деяким кутом до горизонталі.

Показано, що якщо прийняти кут нахилу цієї лінії рівним 45 градусів, то результати виходять досить точними. Еквівалентна висота перерізу визначена як середнє значення між висотою над похилою тріщиною і повною висотою перерізу. Еквівалентна крутильна жорсткість елемента на ділянці похилої лінії приймається рівною жорсткості елемента з умовно постійною жорсткістю при постійній висоті, рівній еквівалентній.

Показано також, що розрахунок за запропонованою методикою відрізняється від розрахунку елемента з нормальною тріщиною тільки тим, що в лівій частині в разі похилої тріщини є ділянка з реальним нахилом перерізу. Інша частина завдання ідентична. Наведено порівняння розрахунків за запропонованою методикою з даними розрахунку в програмі Ліра із застосуванням об'ємних кінцевих елементів. Порівняння показало хороший збіг даних. Запропонована методика розрахунку дозволяє визначити переміщення в похилій тріщині без використання програмних комплексів із застосуванням об'ємних кінцевих елементів. Будучи абсолютно простою, методика має достатню для інженерних розрахунків точність.

**Ключові слова:** кручення, похила тріщина, крутильна жорсткість, переміщення в тріщині, двотавровий переріз, принцип Сен-Венана.

## TWISTING RIGIDITY OF REINFORCED CONCRETE ELEMENTS OF I–BEAM SECTION WITH INCLINED CRACKS

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**Abstract.** The article shows that to determine the torsional stiffness of the element, you must first cut the reinforcement at the site of the inclined crack. After dissecting the reinforcement, the mutual displacement of the crack edges should be determined. This problem is the main and most difficult in the general problem of determining the torsional stiffness of elements with both normal and inclined cracks.

The article is devoted to the solution of this most difficult part of the problem – the determination of the mutual displacement of the banks of the inclined crack of the I-beam element. According to the proposed method, the real element with an inclined crack is replaced by an element with different stiffness in sections. Within an inclined crack, the element has a real slope equal to the slope of the inclined crack. In the area behind the apex of the inclined crack, it is hypothesized that the cross-sectional height of the element varies from a height equal to the height of the zone above the crack to the full cross-sectional height. And change of height occurs according to the law of a straight line. This line is inclined at some angle to the horizontal. It is shown that if we take the angle of inclination of this line equal to 45 degrees, the results are quite accurate.

The equivalent section height is defined as the average value between the height above the inclined crack and the total section height. The equivalent torsional stiffness of the element on the section of the sloping line is taken equal to the stiffness of the element with a conditionally constant stiffness at a constant height equal to the equivalent. It is also shown that the calculation according to the proposed method differs from the calculation of an element with a normal crack only in that in the left part in the case of an inclined crack there is a section with a real slope of the section. The rest of the problem is identical. The comparison of calculations by the proposed method with the calculation data in the Lear program using volumetric finite elements is given. The comparison showed a good match of the data.

The proposed calculation method allows determining the displacements in an inclined crack without the use of software packages using bulk finite elements. Being quite simple, the technique has sufficient accuracy for engineering calculations.

**Key words:** torsion, inclined crack, torsional stiffness, movement in a crack, I-beam, Saint-Venant's principle.

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