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To The Issue of Determination the Rigidity Characteristics of Reinforced Concrete Elements with Normal Cracks

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Abstract

Most of the reinforced concrete slab structures are statically indeterminate systems. In these systems the redistribution of internal forces depends on the nature redistribution of rigidities between their separate elements. The presence of cracks significantly affects the change of elements rigidity of reinforced concrete structures. In the plate-ribbed systems, which include bridge structures, ribbed prefabricated and monolithic ceilings, at the moment when normal cracks are wide enough, spatial torsion cracks may be absent. This technique aimed at determination of the torsional rigidity of rectangular section elements with normal cracks widens the spectrum of researches on the strength and deformability of reinforced concrete elements. The goal of the present research is to improve the methodology for determining the movement of a reinforced concrete element of the rectangular cross-section with normal cracks. In this case, the end of the rod is loaded with a torque on the section part.

Keywords: *normal crack; reinforced concrete ribbed slab; torsion; torsional rigidity*

1. Introduction

The category of plate-ribbed systems includes bridge structures, monolithic and prefabricated ribbed overlaps. The normal cracks arise at the edges of these structures cause of bending moments.It often happens that when a sufficiently wide disclosure normal of cracks, the spatial fracture from torsion are not available. At the same time, the load redistribution between contiguous ribs and between separate precast elements overlap depends not only on the bending, but also by torsional rigidity ribs [1].

Experimental research [2] shows that rigidity of ribs prefabricated slabs on the torsion changes during crack formation. It should be noted that, don't pay heed to importance of bending rigidity and torsional rigidity in redistributing forces in statically uncertain systems, a very large number of theoretical and experimental work is devoted to the study of bending rigidity. These works include works [3,4,5,6], and others. Investigation of torsional rigidity involves a very limited number of papers [7].

Existing methods of determination of torsional rigidity [8, 9] concern only the reinforced concrete elements with spatial (spiral) cracks. The torque is applied to the part of the end surface of the rectangular element [10].The main objective in determining the torsional rigidity is calculating the displacements in the end of the rectangular element. If you use the "two-rod" model [10] may suffer the accuracy of calculations. Therefore, the aim of this article is improvement the methods of determination the displacements of the reinforced concrete element of rectangular section with normal cracks. The end element is loaded with torque on the part of the section.

2. Main part

The analytical model is a reinforced concrete girder element of rectangular cross section. Element is divided into blocks with normal cracks, which arise from the action of bending moment. The torque is applied to the top part of the end surface of the girder (zone compressed by bending).

The results are used for numerical and analytical method of research. We used differential equations and methods of differential and integral calculus.

Consider a concrete element with normal cracks (fig. 1).

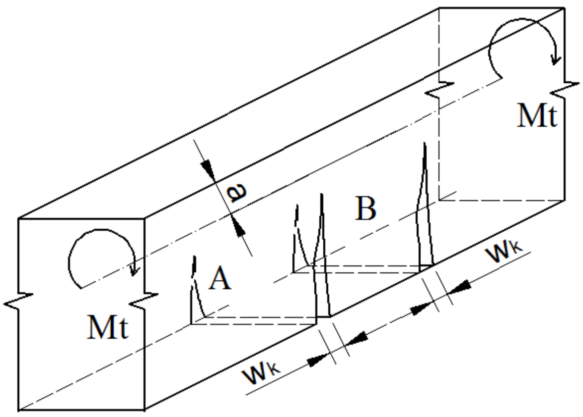

Fig. 1: Scheme of reinforced concrete element with normal crack, which is loaded torque

Transfer of torque from *A* block to block *B* (fig. 1) occurs through the compressed zone of concrete. To determine the rigidity of the reinforced concrete element with normal cracks under the action of torque is required to determine the displacement the block *A* relative to block *B*.(fig. eee)

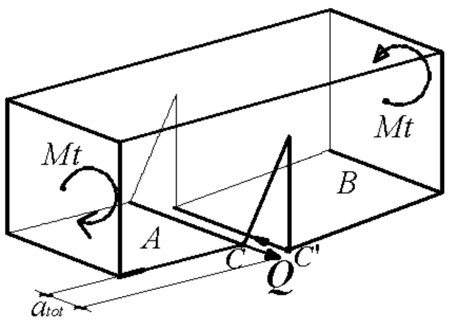

fig. eee Moves of block A relative to block B, separated by a normal crack

Application scheme of torque to the block *B* looks as shown in (fig. 2).

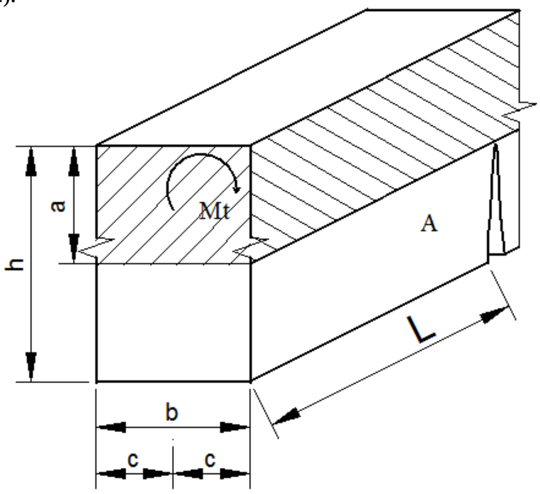

Fig. 2: Scheme of torque transmission through the compressed zone of concrete

Determination of the torsional rigidity of a reinforced concrete element with a normal (from bending) crack can be represented in the following sequence:

1. To create a static definability, one should mentally dissect the longitudinal reinforcement in the fracture.
2. Determine the movement of one block relative to the other. From the compatibility conditions of deformations in the cut-off reinforcement, determine the nudge force Q in it (Figure 0.2). Taking into account the nagel force Q and the external torque Mt, determine the real horizontal displacement in the fracture atot of one block relative to the other (Figure 0.2).
3. Determine the angle of rotation of the fictitious conditionally continuous element φekv as the ratio of the previously determined displacement atot (point 4) to the turning radius, (approximately half the height of the rod section).
4. Determine the torsional stiffness of the element with crack Bt according to the formula:

$$B_t = \frac{M_t \cdot l}{\varphi_{ekv}}$$

To determine the mutual displacement of blocks, it is necessary to determine the stresses in the longitudinal sections of the element, to which a torque is applied to a section of the cross section of the element. **Fig. 2**:

The task of the elasticity theory about torsion rod of rectangular cross section offers a solution based on these assumptions:
- end of the rod is uniformly loaded by the tangential forces;
- the resultant of these forces is the torque Mt;
- according to the method of application torque on Fig. 2 (on a part of the section) stresses and displacements can not be defined by the formulas torsion.

This task can be solved using the finite element method (FEM) using volumetric finite element (FE). There are difficulties in using these elements. Keep in mind that this problem is only part of the solution of the more general task of determining the torsional rigidity of reinforced concrete elements with cracks.
To solve the problem we use the method [10]. The difference will be in the separation the rod into arbitrary number, instead of two.

Consider a rod, cut by a horizontal plane into two linear finite elements – two beams I and II (fig.4). The length of the rod in question L is the length of the block bounded by cracks. The cutting plane passes on the boundary between the elements I and II. In this case, the height of the section of the upper rod is equal to the compressed (from the bend) zone, and the height of the element II is the height of the crack. In Fig. 4 consoles are shown conditionally. In fact, the ends of elements I and II are in the same vertical plane XOZ.

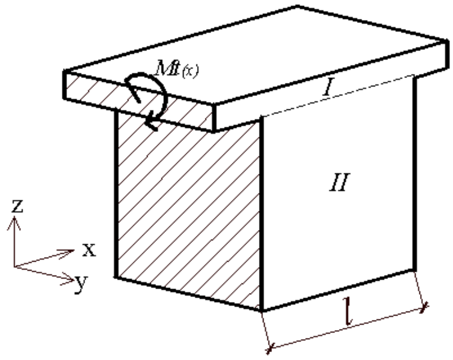

Fig. 4: Scheme of dissection of a rod block into two rods. The block is bounded by cracks on both sides

The forces S (x) act in the plane of dissection in the vertical plane (fig. 5) and tangents τ (x) – in the horizontal plane. The tangential forces are directed along the *y*-axis in Fig. 3a. It should be noted that tangential forces directed along the *x* -axis will act in the plane of dissection, but they will be neglected because of their smaller magnitude in the approximate method under consideration.

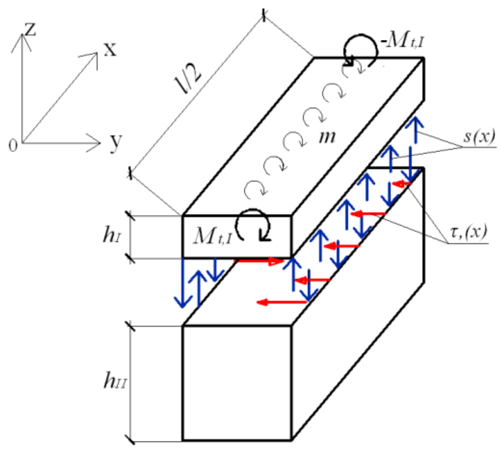

Fig. 5: Efforts acting on the plane of dissection double-layer console rod.

The block, separated by cracks, is represented in the form of a physical model (fig.4). It can be seen that the fibers of such a "model" under the action of torque, as shown in Fig. 4 are subjected to compression-expansion deformations in the vertical and horizontal directions. Volumetric diagrams of linear vertical (transverse) forces and shear forces acting in the horizontal plane of the rod dissection in Fig. 5 are shown in Fig. 6.

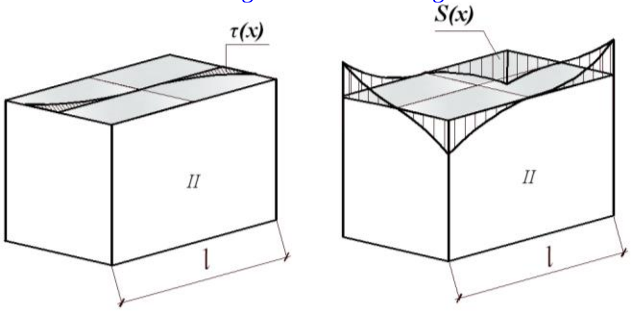

Fig. 6: Diagrams of tangential shear and transverse forces acting in the plane of the dissection of the rod

It should be noted that the dissection of a block of length l parallel planes in the XOY plane into the nth number of layers (rod finite elements) gives greater accuracy of the calculation results. The more number of elements of dissection will be accepted, the higher the accuracy of determining unknown forces and deformations.
Loading scheme of the block *B* (fig. 1) and its division into separate lanes is as shown in Fig. 3.
We spend ***n*** horizontal sections which are parallel to the plane OXY (fig. 3) and get ***n + 1*** lanes (rods).

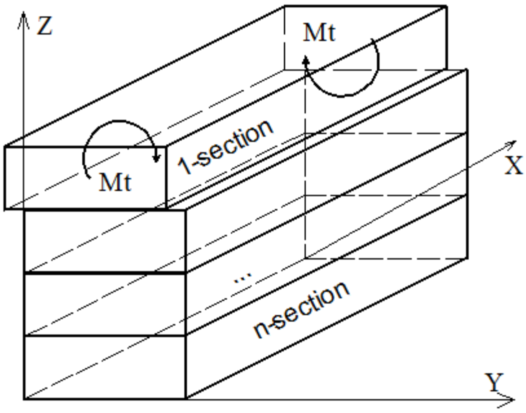

Fig. 7: The scheme of dividing the block into individual strips(rods)

Considering the symmetrical loading block of Fig. 3, the scheme of loading the *i- rod*, bearing in mind the analogy with [10] can be represented as shown in Fig. 4.
The origin of vertical efforts $S_i(x)$ and the tangential $\tau_i(x)$ is explained in [10]. So additional explanations are not going to give. So here we are not going to give additional explanations.

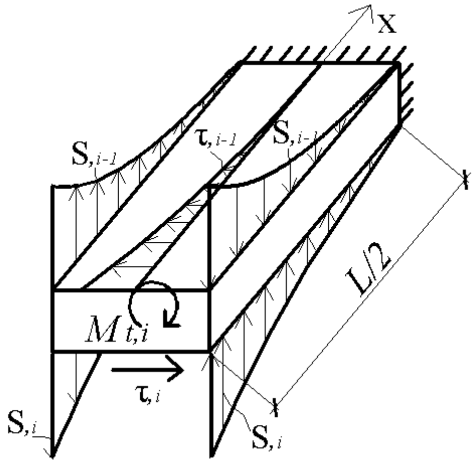

Fig. 8: Scheme of the internal forces applied to the i-rod

Unknown $S_i(x)$ and $\tau_i(x)$ is determined by the joint of deformations community in the i-section (similar to [10]). A typical strings of the system of equations to determine the unknown efforts will look like:

$$\left.\begin{aligned}
&-M_{t,i}\cdot\frac{r_i}{GJ_i}+T_{i-1}\cdot\frac{r_i^2}{GJ_i}+T_i\cdot\frac{r_i^2}{GJ_i}-QS_{i-1}\cdot\frac{b\cdot r_i}{GJ_i}+\\
&+QS_i\cdot\frac{b\cdot r_i}{GJ_i}+T_i''\cdot\frac{r_i^2}{G\cdot b}=M_{t,i+1}\cdot\frac{r_{i+1}}{GJ_{i+1}}-T_i\cdot\frac{r_{i+1}^2}{GJ_{i+1}}-\\
&-T_{i+1}\cdot\frac{r_{i+1}^2}{GJ_{i+1}}+QS_i\cdot\frac{b\cdot r_{i+1}}{GJ_{i+1}}-QS_{i+1}\cdot\frac{b\cdot r_{i+1}}{GJ_{i+1}}-\\
&-T_i''\cdot\frac{r_{i+1}}{G\cdot b};\\
&M_{t,i}\cdot\frac{C}{GJ_i}-T_{i-1}\cdot\frac{r_i\cdot C}{GJ_i}-T_i\cdot\frac{r_i\cdot C}{GJ_i}+QS_{i-1}\cdot\frac{b\cdot C}{GJ_i}-\\
&-QS_i\cdot\frac{b\cdot C}{GJ_i}-QS_i''\cdot\frac{r_i}{EF}=M_{t,i+1}\cdot\frac{C}{GJ_{i+1}}-T_i\cdot\frac{r_{i+1}\cdot C}{GJ_{i+1}}-\\
&-T_{i+1}\cdot\frac{r_{i+1}\cdot C}{GJ_{i+1}}+QS_i\cdot\frac{b\cdot C}{GJ_{i+1}}-QS_{i+1}\cdot\frac{b\cdot C}{GJ_{i+1}}+QS_i''\,\frac{r_{i+1}}{EF};
\end{aligned}\right\}\qquad(1)$$

System (1) shall be compiled for each «k» the seam (longitudinal section).
Consequently, the number of equations is equal to *2n*, where *n* is the number of sections (fig. 7).
In the expression (1) is indicated:
$T_i=T_i(x)$ – the summary tangential efforts which relate with the linear tangential efforts $\tau_i(x)$ by the differential ratio:

$$T_i^I(x)=\tau_i(x) \qquad (2)$$

$QS_i=QS_i(x)$ – the summary vertical efforts which relate with the linear tangential efforts Si(x) by the differential ratio:

$$QS_i^I(x)=S_i(x) \qquad (3)$$

r_i – half the thickness of the i-th rod;
b – width of the cross section of the rod (see fig. 2.);
$C=b/2$ – rod turning radius (half the width of the section);
GJ_i – torsional rigidity of i-th rod
EF – rigidity of conditional rods of unit width, which simulate compression (stretching) the fiber of rods in the vertical direction [10].
The system of equations (1) can be conveniently solved by decomposing the unknown members in the Fourier series by cosines:

$$T=\sum_{n=1}^{\infty}T_n\cdot Cos(\alpha\cdot x),$$
$$QS=\sum_{n=1}^{\infty}Q_n\cdot Cos(\alpha\cdot x), \qquad (4)$$

where $\alpha=\pi\cdot n/l$.

To solve a system of equations, external torque M_t also is decomposite in series by cosines:

$$M_t(x)=\sum_{n=1}^{\infty}M_{tn}\cdot Cos(\alpha\cdot x), \qquad (5)$$

where $M_{t,n}$ – the Fourier coefficient, which indicates the decomposition of the external moment in the Fourier series. This Fourier coefficient is determined simply enough. The character of change of function of the external torque along the length of rod does not affect its definition.
Then it is necessary differentiate, expanded, reduced to the $Cos(\alpha x)$ all unknown and load terms in Fourier series. So, instead of differential equations get a system of linear algebraic equations.
In the case of the same cross-section of all t rods (when the rod is divided into separate layers of equal thickness) will be:

$$\left.\begin{aligned} &T_{i-1}\cdot\frac{r_2}{GJ}+T_i\left(\frac{2r^2}{GJ}+\frac{2\alpha^2 r}{G\cdot b}\right)+T_{i+1}\frac{r^2}{GJ}+QS_{i-1}\left(-\frac{b\cdot r}{GJ}\right)+\\ &+0+QS_{i+1}\frac{b\cdot r}{GJ}=\frac{r}{GJ}(M_{t,i}+M_{t,i+1});\\ &T_{i-1}\left(-\frac{r\cdot C}{GJ}\right)+0+T_{i+1}\frac{r\cdot C}{GJ}+QS_{i-1}\frac{b\cdot C}{GJ}+\\ &+QS_i\left(-\frac{2bC}{GJ}-\frac{2\alpha^2 r}{EF}\right)+QS_{i+1}\frac{b\cdot C}{GJ}=\frac{C}{GJ}(M_{t,i+1}-M_{t,i}); \end{aligned}\right\}\qquad (6)$$

where $r_{i+1}=r_i=r$; $GJ_i=GJ_{i+1}=GJ$.
In the expression (6) through T_i and QS_i are designated unknown expansion coefficients in the Fourier series of cosine (4), and through $M_{t,i}$ – the Fourier coefficients in the expansion of external torque M_t moments in the series (5).
The system of equations (6) is solved ***m*** times, where ***m*** is the upper limit of the summation of series (4) and (5). Usually 7–9 odd term numbers are enough to provide acceptable calculation accuracy.
After determining efforts *Ti(x)* and *QSi(x)* a rod is considered as loaded by external torque *Mt,i(x)* and the efforts $T_{i-1}(x)$, *Ti(x)*; $QS_{i-1}(x)$, *QSi(x)*, defined by solving the equations system (fig. 8).
If known the efforts *Ti(x)* and *QSi(x)*, it is easy to determine the upper part of the block movements relative to its lower part (fig. 1).
After determining the unknown forces T (x) and S (x), determine the angle of rotation of the upper part of the block relative to its lower part. The rotation of the block to which the torque Mt is applied relative to the adjacent block is resisted not only by the compressed zone (not cracked section), but also by the reinforcement. of the block in question along the X axis relative to the adjacent block (Figure 0.18).ΔConsider the displacement

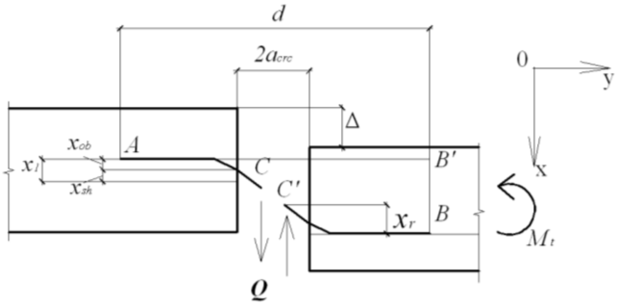

Fig.0.18: Scheme of deformation of reinforcement and mutual rotation of blocks

In Fig. 2.18 the following designations have been adopted:
- *2acrc* – the width of the crack;
- *Xsh* – the displacement of the cutting point of the reinforcing bar from the shift of the latter (in general, from shear and bending, but in view of the small value of acrc, the shear deformations predominate in the main);
-*Xob* – the displacement from the caving of concrete from the consideration of the work of reinforcement in concrete as a rod on an elastic foundation, the role of which is performed by a concrete shell.

After that the unknown transverse (nagel) force *Q* is determined in the armature of element like [10] and [2].
It is determined from the condition that the horizontal *C* and C^I point's displacements (fig. 5) are equal in a place of mental cutting an armature.

$$Q=\frac{a_{Mt}^{ver}-a_{Mt}^{nig}}{2\cdot a_{ob,ed}+2\cdot a_{sh,ed}+a_{Q,ed}^{ver}-a_{Q,ed}^{nig}},\qquad (7)$$

where we have marked:
$a_{ob,ed}$; $a_{sh,ed}$ – the displacements from the concrete crimping and armature shear from the single force $\overline{Q}=1$ action; these displacements are defined as the displacements of the rod, which is based on a continuous elastic base [10];

a_{Mt}^{ver} – the point *C* displacement from the torsion of the upper part, i.e. of the compressed zone (fig. 9) by external torque M_t considering the internal forces $QS_i(x)$ and $T_i(x)$;
$a_{Q,ed}^{ver}$ – the point *C* displacement from the torsion of the upper part, (fig.9) by external torque, generated by a single force in the armature $\overline{Q}=1$;
a_{Mt}^{nig} – the point C' displacement, i.e. the lower part in Fig. 9, from the action of internal forces $QS_i(x)$ and $T_i(x)$, that arise as a result of torsion by a external moment M_t;
$a_{Q,ed}^{nig}$ – the point C' displacement, i.e. the lower part in Fig. 7, from the action of internal forces $QS_i(x)$ and $T_i(x)$, that arise as a result of torsion by a single force $\overline{Q}=1$.

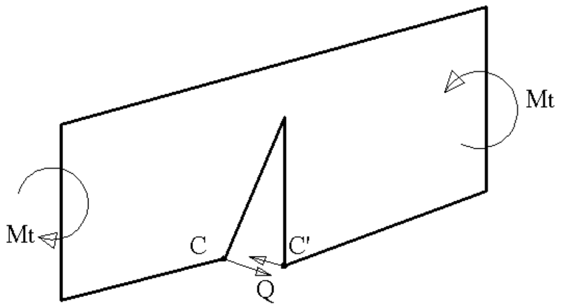

Fig. 9: The scheme of the mutual rotation of two blocks, separated by crack

The components of the displacement in the expression (7), are defined as in [2,10] but with the changes, wich connected with the definition of internal efforts, made in this article.
After calculating the unknown quantity *Q* you can determine the real displacement in the crack w_{tot}.
Order to determine torsional rigidity of the element with normal crack should identify the rotation angle of conventionally continuous (without cracks) element:

$$\varphi_{ekv}=\frac{w_{tot}}{h/2}.\qquad (8)$$

The ratio of the rotation angle of continuous element without cracks to an equivalent, which defined by (8), gives us the ratio of the continuous element rigidity to the rigidity of the element with normal crack.
Use of a multilayer scheme (fig. 3) advantageously differs from a two-layer scheme [10], because accuracy of determining the efforts grows just as in FEM with decreasing the finite element size is increased the accuracy of the result.
Thus, for a girder with cross section *b* x *h* = 10 x 20 cm, with block length between cracks L = 20 cm and with depth of the compressed zone 4 cm, the maximum effort value *S (x)* in the end of the element (in the cross section with a crack) for the multilayer scheme (when the number of layers is equal to five) is constituted to 36,8 N/cm, and for the two-layer scheme – 28,3 N/cm.
As you can see, the difference in values is a significant. Layers thickness (conditionally –dimensions of finite elements), which are needed to obtain an acceptable accuracy, can be determined by trial calculations.

3. Conclusions

A method for determining the internal forces in rod element was developed. Torque is applied to a portion of the cross section. The calculation of these efforts allows to determine the displacements in the cross section with normal to crack under torque action.
This, in turn, allows to determine the torsional rigidity of reinforced concrete element with normal cracks.
The perspective is the development of methodology for determining the rigidity characteristics of concrete elements of an arbitrary cross-section with normal cracks.

In addition, it should develop a program on the computer for automatic calculation of displacements (and rigidity parameters) of elements with normal cracks. In the future, the program will be used as a subprogram in the calculation of bridges, slabs and other ribbed systems taking into account the spatial work.

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