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Consideration of the Torsional Stiffness in Hollow-Core Slabs' Design

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Abstract. The paper discusses the principles of precast concrete hollow-core slabs taking into account their spatial work. It is shown that consideration of spatial work makes it possible to determine the forces in individual floor slabs significantly more precise. The fact that strain redistribution between precast floor slabs depends on slabs' bending and torsional stiffness is shown. The research has been mostly devoted to determination of the bending stiffness with regard to formation of cracks and the change in torsional stiffness, especially considering the presence of normal cracks, which is still unstudied. This paper presents the technique for determining the torsional stiffness of hollow-core slabs with normal cracks. In order to determine the components included in the resolving system of equations, it is proposed to use an approximation method based on the processing of numerical data using spatial finite elements.

1. Introduction

Problem statement. It is known that floors and coverings are the most responsible and material-intensive elements of a building. They perform the most important functions: carry payloads and ensure the building spatial stiffness. Reinforced concrete floors are made in precast (hollow-core, ribbed, flat slabs), cast and composite versions.

Reinforced concrete hollow-core slabs have been made in Europe for over 60 years. During these years they were one of the most used floor elements in residential and public buildings, especially those implemented in the large panel technology. Currently, prestressed hollow-core slabs are one of the most widely applied precast elements in the modern building industry all over the world [1]. Due to the possibility of work under various support or loading conditions, those elements repeatedly work under complex stress state [2].

Those elements are most often designed as simply supported elements, and the computational analysis of such a design are based on the assumptions of plane stress state. In most cases it is supposed, that the floor slabs are under the action of loading uniformly distributed over the whole surface of the slab. Even in case of occurrence of linear or concentrated loads. Hollow-core slabs, although constructed from precast elements, might be treated as floor plate, with the possibility of redistribution of loads to the adjacent precast elements. Such a performance of the floor is possible due to monolithisation of the structure, by casting the concrete in longitudinal joints between slabs or (in many structures) by casting the structural concrete topping layer [3].

The assumption of the plane stress state in the cross-section of the slab is justified, when the element is subjected to uniformly distributed loadings over its entire surface and is supported on the two parallel, relatively rigid supports, e.g. walls or beams with large cross-section. In many cases, this assumption does not reflect the actual conditions of work and behavior of the element. If only one of the elements of the floor structure is supported or loaded in the non-uniform way, then the distribution of the transverse force through the joint of the slabs may cause torsion of the adjacent slabs. The following slabs demonstrate the influence of torsional moments [4]:

a) elements supported on three edges, e.g. corner slab, with longer edge supported on the wall,
b) elements loaded with concentrated force in the longitudinal edge area, e.g. force from the trimmer beam, acting as the support of the neighboring beam, in the area of the large opening or recess cut for staircases and passageways purposes,
c) elements supported with one corner on the column,
d) slanted slabs in the support area, causing the lack of parallelism of the opposite,
e) elements supported on slender beams with relatively large deflections, i.e. Slim Floor type [5].

Accounting for the spatial work of the reinforced concrete overlaps provides significant savings in materials and essentially improves the accuracy of the forces acting in the floor elements determining [7, 15, 17].

The precast hollow-core slabs use is determined by high strength and stiffness, a small values of a section height and reduced thickness, sufficient sound insulation, high factory readiness, a smooth ceiling, etc.

Combining precast slabs into a diaphragm is performed by the joint embedment and embedded parts welding, when various splines (round, longitudinal, open, etc.) are arranged at the ends on the side surfaces of precast slabs.

Calculation of precast overlaps and covering slabs in traditional design is conducted as for the beam structures working on transverse bending.

Calculation of the hollow-core slabs for bending with torsion is not performing in traditional design, although as a result of spatial work in individual slabs torsional moments arise in addition to bending moments.

Various methods for the spatial calculation of floors have been developed. These methods can be divided into five groups:

1. Calculation of pressure on the main beams, taking into account the coefficients of the transverse installation (lever method, eccentric compression and elastic supports).
2. Replacing of span structure with beam grillage.
3. Replacing of span structure with orthotropic slab.
4. Dividing the structure into separate elements with subsequent consideration of the work of each of them and setting up conditions for the displacement compatibility.
5. Numerical calculation methods with software systems application, such as ANSYS, Nastran, Abaqus, Lira etc.

Results of calculations by numerical methods are closest to reality. However, they have some disadvantages, among which is quite complex accounting for changes in the stiffness parameters of slabs as a result of cracking. The calculations according to p. 1-3 were applied relatively a long time ago and are not acceptable at the present.

In our opinion, the most acceptable are discrete-continuum methods (p. 4). B.E. Ulitsky [17] can be considered as their founder. P.F. Drozdov [12], was the first who obtained an exact solution to the problem of the discrete-continual model in a particular case, which allows to obtain a solution in closed form in some cases. Unknown vertical reactions are determined from system of differential equations of the second order solution:

$$\begin{aligned}&\frac{1}{EI_{k-1}}(p_k - p_{k-1} - q_{k-1}) - \frac{1}{GI_{k-1}}\left(\frac{b_{k-1}}{2}\right)^2 (p_k^{II} + p_{k-1}^{II}) = \\ &\quad = \frac{1}{EI_k}(p_{k+1} - p_k - q_k) + \frac{1}{GI_k}\left(\frac{b_k}{2}\right)^2 (p_k^{II} + p_{k+1}^{II})\end{aligned} \tag{1}$$

where p_k are vertical strains of interaction of slabs with each other in the k^{th} joint of floor. At the same time, the monolithic joint is modeled by a cylindrical hinge, transmitting only vertical interaction forces from slab to slab; q_k is the load on k^{th} slab; EI_k and GI_k are respectively bending and torsional stiffness of k^{th} slab; b_k is the width of k^{th} slab.

However, the method of P.F. Drozdov is acceptable only for the calculation of the particular case when only vertical interaction forces are taken into account between the slabs. Therefore, it can only be used for the calculation of floors consisting of precast hollow-core slabs. In addition, this method does not account for bending of precast slabs in the transverse direction and shear of the monolithic joint. Experimental studies [10] prove that these factors influence on the redistribution of forces between individual floor slabs and should be taken into account in the calculations.

T.N. Azizov derived a general system of differential equations for calculating the floors by the discrete-continuous method. According to this method, the floor is cut into separate linear elements (precast slabs, a strip of T-shape or solid cast slab, etc.). In the general case, four unknown functions of forces act on the i^{th} cutting line: tangential forces T_{i-1} and T_i, vertical linear forces S_{i-1} and S_i, linear transverse bending moments M_{i-1} and M_i, horizontal linear traverse forces H_{i-1} and H_i. As a result, a system of *4n* differential equations with *4n* unknown force functions was derived in [7]. As a result, the P.F. Drozdova, B.E. Ulitsky, A.S. Semchenkov calculation method is a particular case of the method [7].

Calculations of floors considering for the spatial work can significantly more accurately determine the forces occurring in the floor. The T.N. Azizov, P.F. Drozdov system of equations includes both bending stiffness of individual elements (for example, precast hollow-core slabs) and their torsional stiffness, which are changing after the various crack formations. Consequently, the redistribution of forces in floors (and other complex statically indefinable systems, such as frameworks of multistoried buildings, grillages and cross-beam systems) substantially depends on both the bending stiffness of their elements and the torsional stiffness.

Taking into account that the portion of torsional moments is mainly (especially at the initial stages of loading) relatively small, only normal cracks appear in the floor elements. Spiral torsion cracks as well as inclined cracks are mostly absent. In this case, the question arises, does the bending stiffness of the element with a normal crack change? If so, how much?

As for the calculation of the bending stiffness of a reinforced concrete element with cracks, this issue has been studied in sufficient detail [11, 23, 24]. Much less work is devoted to issues of torsional stiffness of reinforced concrete elements with cracks. Moreover, all these works are devoted to the determination of the stiffness characteristics and strength of elements with spiral (spatial) cracks [13, 22]. A very limited number of works [6, 9, 19] is devoted to the calculation of the torsional stiffness of reinforced concrete elements with normal cracks. However, even in these works little attention was paid to taking into account the nonlinear properties of concrete when determining the torsional stiffness of the rod element. In addition, these works do not take into account the possibility of calculating reinforced concrete elements with normal cracks with multi-row reinforcement like in the hollow-core slabs.

In this connection, *the purpose of this paper* is to study the effect of calculation taking into account the spatial work of the slabs on the redistribution of forces between them, the development of a method for determining the torsional stiffness of reinforced concrete hollow-core slabs with normal cracks, taking into account the multi-row arrangement of reinforcement, as well as setting research objectives in the direction of the floors made of precast hollow-core slabs design.

2. Presentation of the Conducted Studies

2.1. Calculation of precast floors taking into account the spatial work (the mutual interaction of the slabs). At the floors consisting of precast hollow-core slabs calculation, it is assumed (similar to [7, 12]) that slabs are conventionally connected along longitudinal sides with cylindrical hinge. That is, only vertical forces of their interaction are transmitted from the slab to the slab. In this case, one can use the abbreviated system of equations [7], where out of the four unknown force functions, only the vertical forces $S_i(x)$ remain. In addition, unlike the P.F. Drozdov system (1) bending of the slabs in the transverse direction, as well as the shift of the monolithic joint connecting the plates with each other should be taken into account. This can be seen from Figure 1, where a diagram of the deformation of the floor fragment cross section consisting of hollow-core slabs is demonstrated.

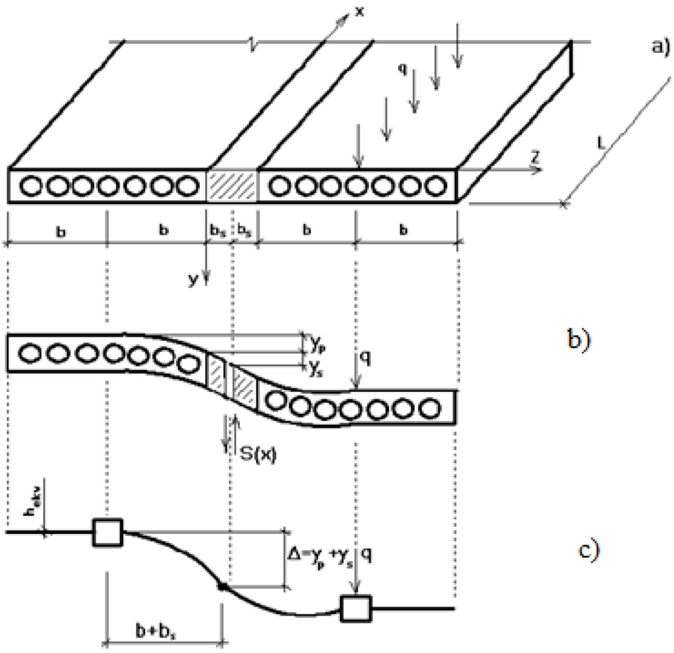


Figure 1. Floor design diagram taking into account slabs bending in transverse direction and monolithic joint shift:
a) real diagram; b) floor cross section deformation diagram; c) design diagram

The cross-section of the floor with a monolithic joint is deformed, as shown in Fig. 1, i.e., the cross section is twisted and the joint is shifted. When modeling the floor in the form of a rib with a shelf at the center of gravity level (Fig. 1, c), the bend of the shelves will simulate both the bend of the slab and the joint shift. The stiffness of the shelves of such a system for bending in the transverse direction must be chosen so that the movement of the end of the shelf in the hinge Δ is equal to the total displacement Y from the bending of the cross section of the slab Y_p in the transverse direction and the joint shift Y_s (Fig. 1). The line of the system of differential equations for such a case has the form (2), and the equivalent thickness of the conditionally constant thickness of the shelf (in Fig. 1, c) can be determined by expression (3):

$$-\frac{1}{EI_i}\cdot MS_{i-1}+\left(\frac{1}{EI_i}+\frac{1}{EI_{i+1}}\right)\cdot MS_i+\frac{1}{EI_{i+1}}\cdot MS_{i+1}+\frac{L_i\cdot R_i}{GI_i}\cdot MS''_{i-1}+\left(\frac{R_i^2}{GI_i}+\frac{L_{i+1}^2}{GI_{i+1}}\right)\times$$
$$\times MS''_i+\frac{L_{i+1}\cdot R_{i+1}}{GI_{i+1}}\cdot MS''_{i+1}+\left(\frac{R_i^3}{3D_i}+\frac{L_{i+1}^3}{3D_{i+1}}\right)\cdot MS_i^{iv}=\frac{1}{EI_{i+1}}\cdot MQ_{i+1}-\frac{1}{EI_i}\cdot MQ_i \qquad (2)$$

$$h_{ekv}=\sqrt[3]{\frac{4(1-\mu^2)\cdot(b+b_s)^3}{E\left(\frac{b^3}{3D}+\frac{b_s}{G_s\cdot h_s}\right)}} \qquad (3)$$

where μ is the Poisson's ratio of floor material; h_s is the monolithic joint thickness (spline thickness); G_s is the shear modulus of concrete of monolithic joint.

Thus, the system of precast slabs interconnected by monolithic joints is reduced to a system of ribbed slabs with shelves located along the centers of gravity of the ribs (in Fig. 1, c). Equations (2) are compiled for each i^{th} joint between precast hollow-core slabs. The MS_i and MQ_i in (2) denote the functions of bending moments from, respectively, the unknown forces S_i acting in the i^{th} joint and the external load q^i acting on the i^{th} slab. D_i is the cylindrical stiffness of the slab at bending in the transverse direction. L_i and R_i are respectively distance from the center of gravity of i^{th} slab to the monolithic joint on the left and on the right. EJ_i and GJ_i are respectively bending and torsional stiffness of i^{th} hollow-core slab.

It is convenient to solve the system of differential equations (2) using the unknown functions expansion $MS_i=MS_i(x)$ in Fourier series in sines. However, it is very difficult and inconvenient to simulate the conditions for supporting the ends of slabs that differ from rigid supports (for example at the support by flexible girth rails and beams). Despite the fact that it does not greatly affect the overall work of the floor, with sufficiently flexible beams by which the slabs are supported, the flexibility of the slab supports should be taken into account. In this case, the system of equations (2) is better solved numerically, which is the subject of further research, as will be discussed below. At the same time, in most cases the slabs are supported by rigid supports (walls, rigid girth rails) and the use of the system of equations (2) with its solution using the decomposition of functions into Fourier series is quite acceptable.

As has been proved in systems of equations (1) and (2), in order to determine the forces of mutual interaction of the slabs, both bending *EI* and torsional *GI* stiffness of the slabs are need.

The advantage of the floors calculation taking into account the spatial work, as well as the fact of the significant torsional moments (as opposed to the traditional design, when each hollow-core slab is calculated as a beam element, working only on bending load) occurrence will be shown by a simple example. Let's suppose there is a fragment of the floor consisting of five hollow-core slabs with a width of 1200 mm and a span of 6000 mm. In addition, one extreme slab (extreme to right) is supported on the longitudinal (long) side by the wall. The floor is loaded with the uniformly distributed load of 8 kPa. The scheme of such floor is shown on Fig. 2.

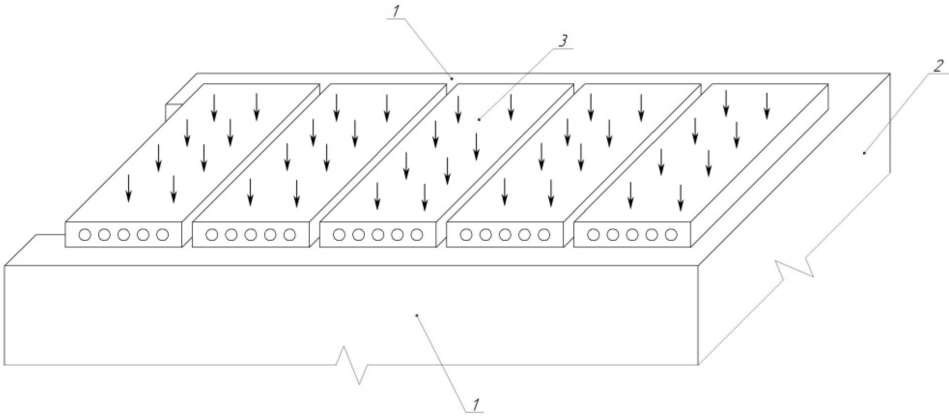
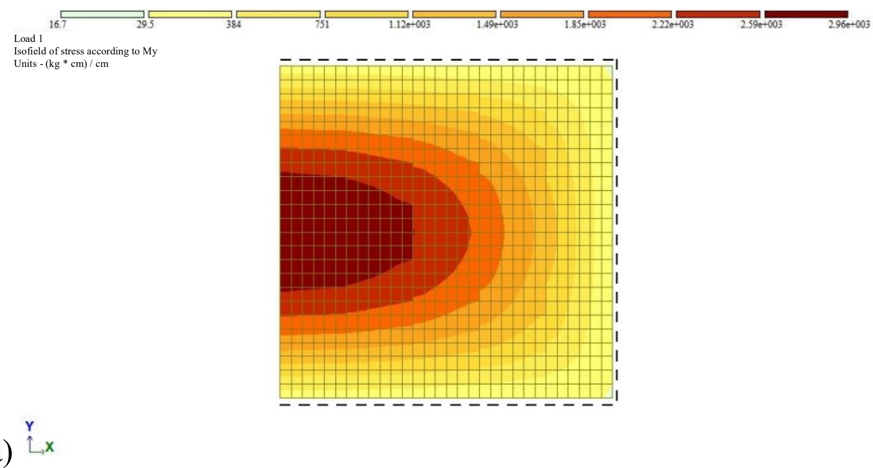


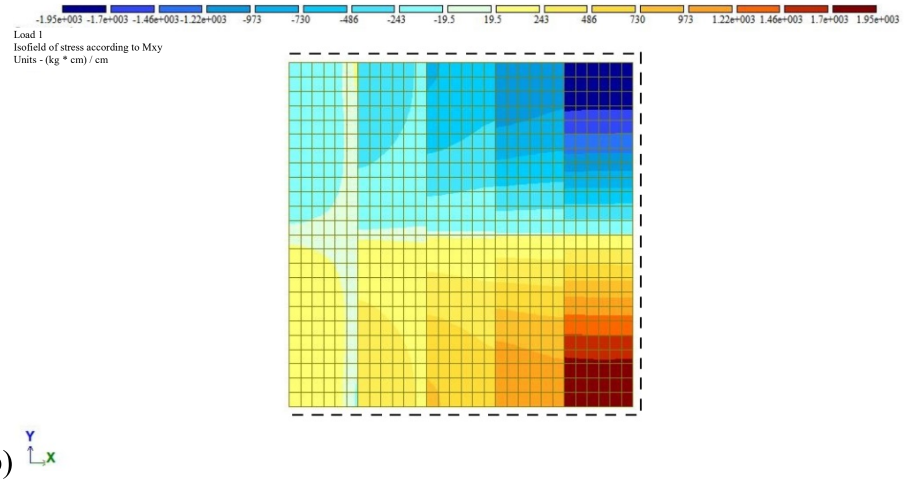
Figure 2. Scheme of floor of five hollow-core slabs, the extreme right slab of which is supported on the longitudinal side by wall. 1 – end-walls; 2 – longitudinal wall; 3 – slabs.

On the Fig. 3 the distribution of bending and torsional moments is demonstrated.

Even if the floor slabs are not supported by the longitudinal (long) sides and a local increased load acts on one of them, the forces in the overlap are significantly redistributed and both bending and torsional moments occur in the slabs. Let's show it by example. Suppose there is a floor of the same (as in the previous example) slabs, which are supported only on the ends. All slabs are loaded with a uniformly distributed load of 5 kPa, and the middle (third) slab is loaded with a local load of 20 kPa. Such local loading can simulate the availability of equipment, storage of materials, etc.



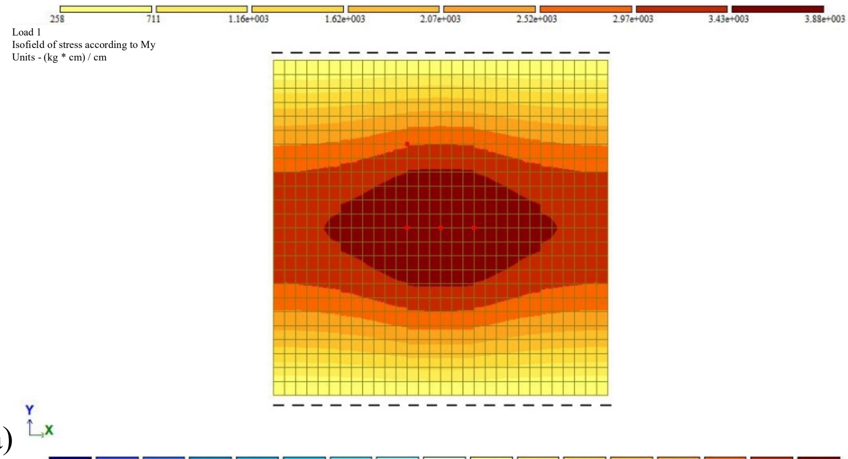
a)



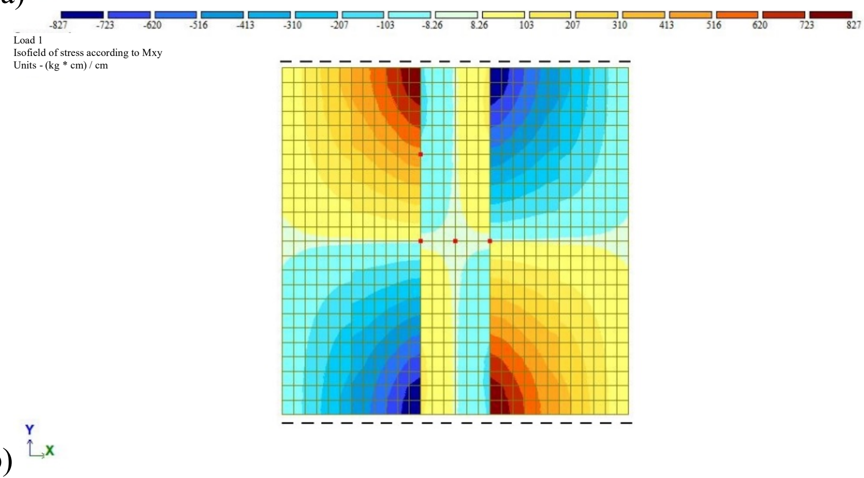
b)

Figure 3. Distribution of bending (a) and torsional (b) moments in the fragment of floor. The dashed lines in the figure indicate the slab's supports

The Fig. 4 presents the distribution of bending and torsional moments in the fragment of floor consisting of five hollow-core slabs. Calculations were performed in Lira software.



a)



b)

Figure 4. Distribution of torsional (a) and bending (b) moments in the fragment of floor. The dashed lines in the figure indicate the slab's supports

It has been established by calculations (Fig. 4) that sufficiently large bending and torsional moments act in a section of about a quarter of the span. However, the magnitude of the torsional moment is not enough for the formation of spatial torsional cracks. At the same time, in these same sections, bending moments are sufficient for the formation of normal cracks. In such sections, normal cracks are formed, but torsional moments will act. Consequently, in such sections, the torsional stiffness of the slab will change, which in turn will entail a redistribution of forces between the individual slabs from which the floor fragment is composed.

Thus, using simple examples, in a fragment of a floor of hollow-core slabs, which is usually calculated according to the beam scheme it has been proved, that not only bending moments, but also torsional moments arise. In addition, considering that in the slabs mostly normal cracks appears, but there are also torsional moments in the cross sections, one should have a technique allowing to calculate the torsional stiffness of reinforced concrete slabs with normal cracks.

The method for determining the torsional stiffness of reinforced concrete hollow-core slabs with normal cracks is considered below.

2.2. General provisions in determining the torsional stiffness of hollow-core slabs with normal cracks. After formation of normal cracks, the torsional moments are transmitted from block A to block B (Fig. 5) through the uncracked part of the concrete and the longitudinal reinforcement. As a result of the presence of cracks, two adjacent blocks separated by a crack are displaced by Δ, which can be decomposed into a horizontal Δx and vertical Δz components.

Torsional stiffness B_t of reinforced concrete element with normal crack can be determined by formula:

$$B_t = B_0/k_{crc} \tag{4}$$

where B_0 is the initial stiffness of element without cracks, determined by the known formulae of the resistance of materials; k_{crc} is the coefficient (k_{crc}>1), representing the ratio of the deformability of an element with a normal crack to the deformability of an element without cracks.

This coefficient can be easily determined (see Fig. 5) by the formula:

$$k_{crc} = (\Delta + \Delta_0)/\Delta_0 \tag{5}$$

where Δ_0 is the displacement from the torsion of an element without cracks, which is determined by the well-known formula of the resistance of materials as the rod angle of rotation multiplied by the distance to the point in question (in figure 4 it is point *C*).

Thus, the coefficient k_{crc} is equal to the ratio of the absolute displacement of the point C^I to the absolute displacement of the point *C*, i.e. this is a coefficient that shows how many times the stiffness of an element without cracks is greater than the stiffness of an element with a normal crack.

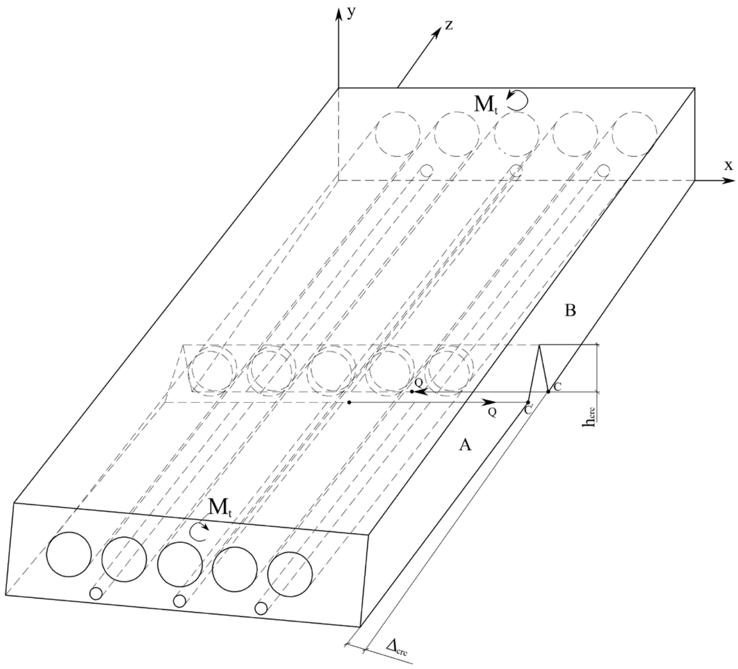


Figure 5. Diagram of displacements in a section with a normal crack and forces in the reinforcement of a hollow-core plate subjected to torsion.

From expressions (5) it can be seen that if the displacement Δ of two blocks separated by a normal crack (see Fig. 4) is found, then the torsional stiffness of any element with normal cracks, including hollow-core slabs, will be determined.

2.3. Determination of mutual displacement Δ_{crc} of adjacent blocks. It has been indicated above, that the main problem for determining torsional stiffness is to determine the mutual displacement of two blocks separated by a normal crack. This problem can be solved in various ways, including approximate one [6]. At first glance, the solution of this problem by simulating a hollow-core slab with volume finite elements using the well-known software packages ANSYS, Abaqus, Lira, etc. seems to be the most accurate. However, a detailed analysis turns out to be almost insurmountable obstacles. The most important of them is the correct modeling of the connection between the concrete and the longitudinal reinforcement during its operation on the load perpendicular to its axis. The fact is that when the reinforcement is working on the transverse load, one part of the concrete under the reinforcing bar is crushed, and the opposite part "moves" away from the concrete practically without any resistance (Fig. 6). In addition, it is known that the concrete in the zone of contact with the reinforcement has mechanical characteristics that differ from the characteristics of the main part of the reinforced concrete element.

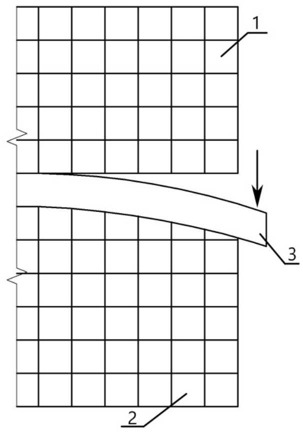


Figure 6. Scheme of the reinforcing rod deformation under the action of transverse load

In this regard, the displacement of the reinforcement from the transverse load should be determined experimentally. In recommendations [14] a formula is given, obtained on the basis of the processing of experimental data, from which it is possible to determine the transverse displacement of a reinforcing bar loaded with a load perpendicular to its axis:

$$a_{loc} = 1000\frac{Q^2}{d_s^3 E_c^2} + \frac{Q}{d_s E_c} \qquad (6)$$

where d_s and E_c are respectively, the diameter of the reinforcement and the Young modulus for the concrete; Q is the force applied to the reinforcing bar in a direction perpendicular to its axis.

When determining the mutual displacement of the edges of a normal crack, a finite element model is proposed to be used as an auxiliary material. The essence of this approach is as follows. First, mentally cut all the bars of the longitudinal reinforcement and determine the movement of one block relative to another in the element with the cut reinforcement. Using the simulation by volume finite elements, it is necessary to determine the mutual displacement Δ_{crc} of the normal crack sides (in the model with cut reinforcement) at the edges (points C and C^I in Fig. 5). On the basis of a series of calculations, it is easy to obtain the function of Δ_{crc} versus the height h_{crc} of a crack $\Delta_{crc}=f(h_{crc})$ by modeling with volume finite elements. It is known that the standard sizes of hollow-core slabs have few variants differing in the number of voids. It can be four, five, six and seven hollows in cross section. Therefore, there will be several functions of the type $\Delta_{crc}=f(h_{crc})$ (how many standard sizes of hollow-core slabs). So, for example, the function for a hollow-core slab 220 mm high, 159 mm hollow diameter and their number equal to six (standard plate 1200 mm wide) will look like:

$$\begin{aligned}
\Delta_{crc,x} &= -0.366h_{crc}^2 + 12.36h_{crc} - 22.531;\\
\Delta_{crc,y} &= 0.057h_{crc}^2 + 1.782h_{crc} + 11.133;\\
\Delta_{crc} &= -0.268h_{crc}^2 + 11.148h_{crc} - 11.593.
\end{aligned} \qquad (7)$$

At that $\Delta_{crc} = \sqrt{\Delta^2_{crc,x} + \Delta^2_{crc,y}}$ is full mutual displacement of the crack sides.

A comment should be made here. Formulae (7) are obtained from a series of calculations with the following materials characteristics and units. h_{crc} – crack height in cm; concrete modulus of elasticity E_b=25000 MPa. Unit torsional moments M_t=1 N·cm is applied to slab. Values $\Delta_{crc,x}$, $\Delta_{crc,y}$ and Δ_{crc} are in mm·10^8. That is, to obtain the value of the displacements in the calculated slab, the values of (7) should be multiplied by the magnitude of the current torsional moment and by the ratio of the slab strain modulus to the strain modulus given here.

Knowing the values of $\Delta_{crc,x}$ and $\Delta_{crc,z}$ it is easy to determine the mutual displacement of the crack sides at the locations of the reinforcement. The distribution of displacements across the width of the section will be linear. This follows from the decision of Saint-Venant, in which the cross-sections are bent (in the direction of the longitudinal axis), but it is assumed that the displacements of points lying in the plane of the cross-sections occurs so that the projection of the deformed section on the plane perpendicular to the axis retains its original shape sections (see, for example, [16]). In other words, the cross-sectional shape does not change, but it is bent in the direction of the longitudinal axis. Fig. 7 shows a diagram of the hollow-core slab cross section rotation (the voids are conventionally not shown).

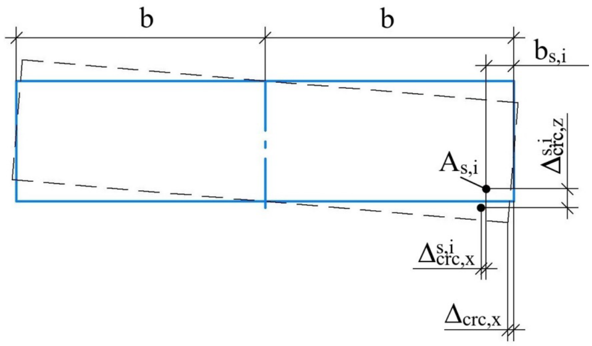


Figure 7. The scheme of the cross-section rotation during torsion and the position of the location point of the reinforcing bars determination

It can be seen from the figure that if the distance $b_{s,i}$ from the slab face to the location of the reinforcement $A_{s,i}$ and the displacement value Δ_{crc} on the plate face is known (determined from the solution of the problem by approximating the results of the finite element calculation using formula 7), then the displacement at the location of the reinforcement $A_{s,i}$ will be determined by the obvious expression:

$$\Delta^{s,i}_{crc,x} = \Delta_{crc,x}\left(1 - \frac{b_{s,i}}{b}\right) \tag{8}$$

In the same way all mutual displacements in the locations of reinforcement $\Delta^{s,i}_{crc,x}$ and $\Delta^{s,i}_{crc,z}$ are determined.

2.4. Algorithm for determining the torsional stiffness of a hollow-core slab with a normal crack. The main task, as has been shown, is to determine the mutual displacement of the sides of a normal crack in a scheme with conventionally cut rods of longitudinal reinforcement. After that, the algorithm for determining the torsional stiffness of a hollow-core slab with normal cracks has the following structure should be applied (the approach is based on the technique [9, 21], but it takes into account the multi-row arrangement of reinforcement).

1. By expressions (7), the mutual displacements Δ_{crc} of the crack sides at the extreme face of the slab from the action of an external moment of magnitude M_t should be determined.

2. By expression (8), the mutual displacement of the crack sides at the locations of all reinforcement bars $\Delta^{s,i}_{crc}$ (i = *1 ... n*, where n is the number of longitudinal reinforcement bars) should be found.

3. The displacement from the local crushing of the concrete under the reinforcing bar from the action of a single dowel force, should be calculated from the expression (6) $Q=1$:

$$\Delta_{loc}=\frac{1000}{d_s^3E_c^2}+\frac{1}{d_sE_c} \tag{9}$$

4. The mutual displacement Δ_{crc}^{Q} of the sides of a normal crack should be calculated from the action of oppositely directed unit forces $Q=1$, applied at the level of longitudinal reinforcement (see. Fig. 4). Moreover, in the model using volume finite elements, the forces $Q=1$ are applied as shown in Fig. 5. As a result, there were obtained the dependences Δ_{crc}^{Q} similarly to (7). That is, the difference between determining the values of Δ_{crc} and Δ_{crc}^{Q} is only in the scheme of torsional moment application (in the first case, in the center of gravity of the end section; in the second, in the level of the reinforcement location by two mutually opposite forces).

5. From the condition of rotation of the section as rigid in its plane (the above Saint-Venant solution) from geometric similarity determine the mutual displacement of the crack sides should be determined at the locations of the $A_{s,i}$ reinforcement using formulae similar to the expression (8) (with the substitution of Δ_{crc}^{Q} instead of Δ_{crc}). Thus, define all the values $\overline{\Delta_{Qi,j}}$ will be found, which are the displacement of the crack sides at the location of reinforcement i from the action of a single force $Q_j=1$ applied at the location of reinforcement j.

6. The unknown dowel forces Q_i in each longitudinal reinforcement $A_{s,i}$ is determined from the strain compatibility condition at the place of their imaginative dissection. These forces are determined from the system of equations:

$$\begin{gathered}
Q_1\left(\overline{\Delta_{Q1,1}}+2\overline{\Delta_{loc}}\right)+Q_2\left(\overline{\Delta_{Q1,2}}+2\overline{\Delta_{loc}}\right)+\cdots+Q_n\left(\overline{\Delta_{Q1,n}}+2\overline{\Delta_{loc}}\right)=\Delta_{crc}^{s,1}\\
Q_2\left(\overline{\Delta_{Q2,1}}+2\overline{\Delta_{loc}}\right)+Q_2\left(\overline{\Delta_{Q2,2}}+2\overline{\Delta_{loc}}\right)+\cdots+Q_n\left(\overline{\Delta_{Q2,n}}+2\overline{\Delta_{loc}}\right)=\Delta_{crc}^{s,2}\\
\cdots\cdots\cdots\\
Q_n\left(\overline{\Delta_{Qn,1}}+2\overline{\Delta_{loc}}\right)+Q_2\left(\overline{\Delta_{Qn,2}}+2\overline{\Delta_{loc}}\right)+\cdots+Q_n\left(\overline{\Delta_{Qn,n}}+2\overline{\Delta_{loc}}\right)=\Delta_{crc}^{s,n}
\end{gathered} \tag{10}$$

After determination of all the dowel forces, it is easy to find the real displacement Δ in the crack:

$$\Delta=2\cdot Q_{max}\overline{\Delta_{loc}} \tag{11}$$

where Q_{max} is the maximum dowel force in the rods of the longitudinal reinforcement, determined from the system (10). Since all reinforcement bars resist the mutual displacement of the normal crack sides, and according to (9) the concrete crushes equally under all the rods, therefore in (11) the maximum value of the dowel force is assumed.

When comparing the total displacement Δ determined by (11) in a fracture with a similar displacement according to [21], where the displacement in a fracture of an element with a single reinforcement is determined, one can see that they differ in that the maximum force Q_{max} in reinforcing bars is in (11), which naturally will be less than the dowel force Q with a single reinforcement. Therefore, the above method for determining the displacement in a crack is significantly more accurate compared to [21].

After determining the value of Δ by (11), the torsional stiffness of an element with a normal crack can easily be calculated by (4) with regard to (5).

3. Conclusions and Intended Ways for Solution of the Pointed Problems

1. Based on the results of research presented in this paper, as well as previous works [6, 9, 20, 21], it can be concluded that, in order to eliminate the effect of normal cracks on reducing the torsional stiffness of hollow-core slabs, the formation of normal cracks should be avoided. If this fails, then the depth of normal cracks should be limited. This can be achieved by using prestressing technology instead of producing reinforced concrete elements. In this case, it is necessary to carry out a spatial calculation of an overlap (according to paragraph 1 of this article), then to determine the sections where cracks form, the torsional stiffness of the slabs, taking into account the presence of cracks, and to repeat the spatial calculation. If, resulting of the introducing of prestressing into

the element, it turns out that the redistribution of forces in the floor is more efficient than in the floor of plates without prestressing (taking into account the cost of both options), then the use of prestressed elements should be recommended. If the slabs without prestressing are more efficient (economically), then it can be abandoned in this case.

Thus, the spatial calculation of the floor and the determination of the torsional stiffness of slabs with normal cracks will allow in each case to make a decision on the need or absence of such prestressing of the plates.

2. The technique presented in this paper in many cases makes it possible to determine the torsional stiffness of hollow-core slabs with normal cracks quite accurately and, as a result, it is more correct for the calculation of the floor with considering for the spatial work. However, in some cases, especially at low concrete strengths, calculations in the elastic stage can lead to errors. In this case, calculations should be carried out to determine torsional stiffness, taking into account the nonlinear properties of concrete. The beginning of such works was laid in [8, 18]. However, these works are not brought to the final result and should be developed for practical application.

3. To use the proposed approximation approach successfully aiming to solve the problem of determining of displacements in an element (generally, for any cross-section) with a normal crack, one should create a database with approximation functions like (7), to enable engineers and designers to use in practice such approximation dependences for any cross-section parameters. This will save engineers in practical design to carry out complex and cumbersome calculations, and focus them on the design and save a lot of time and money.

4. For the system of equations (2), which is solved in the cited papers using Fourier series, a methodology and algorithms for numerical solution should be developed. This will allow solving problems on the calculation of floors with any conditions for supporting the ends of the slabs (on flexible supports, etc.) and any law of the distribution of bending and torsional stiffness along the length of the plates.

5. For automated calculation of floor taking into account spatial work, practical algorithms should be created that take into account all possible factors for changing torsional and bending stiffness, as well as taking into account the drawbacks of using equations like (2) when taking into account the bending and torsional stiffness that vary over the span length of hollow-core slabs.

Thus, this paper not only presents the solution for one of the tasks of the general design problem of calculation of the precast concrete hollow-core slabs but the intended ways of the whole research direction have been outlined.

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