National Committee of Ukraine by Theoretical and Applied Mechanics
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XIX International Conference

DYNAMICAL SYSTEM MODELLING AND STABILITY INVESTIGATION

MODELLING
&
STABILITY

PROCEEDINGS OF CONFERENCE REPORTS
Kiyv, Ukraine

May 22-24, 2019

ВІСНИК
Київського національного університету імені Тараса Шевченка

The various aspects of theoretical and applied researches are represented in proceedings of conference reports. Problems of adequate mathematical model of studied processes are considered.
Problems of control synthesis and stability investigation of movements are separately allocated. Significant numbers of papers are devoted to modeling of economic problems, biological and social phenomena. Big quantity of reports presented at the conference is devoted to the problems of applied mechanics. Logic-mathematical methods of modeling are considered.
Prepared by **C.Sc., Docent** A.V.Shatyrko

Recommended for printing by Scientific Council
of Computer Science and Cybernetics faculty
of Taras Shevcheno National University of Kyiv.
Scientific Editor: Dr.Sc., Prof.
Khusainov D.Ya.
Reviewer: Dr.Sc., Prof.
Boychuk A.A.

В материалах конференции представлены различные аспекты теоретических и прикладных исследований. Рассмотрены вопросы создания математических моделей, адекватно описывающих исследуемые объекты.
Отдельно рассмотрены проблемы синтеза управления и исследования устойчивости движения. Значительное количество работ связано с моделированием экономических, биологических и социальных процессов. Большое количество работ посвящено проблемам теоретической и прикладной механики. Рассмотрены логико-математические методы моделирования.
Подготовлено **канд. ф.-м.н., доц.** А.В.Шатырко

Рекомендовано к печати Ученым Советом
факультета компьютерных наук и кибернетики
Киевского национального университета
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В матеріалах конференції представлено різні аспекти теоретичних та прикладних досліджень. Розглянуто питання створення математичних моделей, що адекватно описують об’єкти.
Окремо розглянуто проблему синтезу керування та дослідження стійкості руху. Значна кількість праць пов’язана із моделюванням економічних, біологічних та соціальних процесів. Велика кількість праць присвячена проблемам теоретичної та прикладної механіки. Розглянуті логіко-математичні методи моделювання.
Підготовлено **канд. ф.-м.н., доц.** А.В.Шатирко

Рекомендовано до друку Вченою Радою
факультету комп’ютерних наук та кібернетики
Київського національного університету
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During the period May 22-24, 2019 the traditional International Conference “Dynamical Systems Modeling and Stability Investigation” was held.

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FACULTY OF COMPUTER SCIENCE AND CYBERNETICS

AND

DEPARTMENT OF COMPLEX SYSTEMS MODELING

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   - Lyapunov's methods in system investigation.

2. **Methods and technologies of computer modeling.**
   - Numerical Methods of Mathematical Physics.
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   - Software and Systems Modeling

3. **Modeling and investigation of processes in mechanics.**
   - Mathematical modeling in composite materials of mechanics.
   - Modeling and investigation of dynamical processes in elastic and hydroelastic systems.
   - Mathematical modeling in connected fields of mechanics.

4. **Method of control and complex systems research**
   - Methods of control and optimization.
   - The continuous-discrete systems
   - Methods of differential games.
   - Fuzzy modeling and systems with uncertainty.
   - Modelling in economy and ecology.

5. **Logic-mathematical methods of modeling.**
   - Methods and tools of subject domains specifications.
   - Methods and tools of software systems description.
   - Modal and temporal formalisms of systems modeling

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APPROXIMATE METHOD OF SOLUTION OF SYSTEM OF A FUNCTIONAL WIENER-HOPF EQUATIONS

Dudyk M.

Key words: Wiener-Hopf equations system, method of approximate solution
AMS Subject Classification: 39B72, 45E10

Many plane boundary problems on mathematical and theoretical physics and applied mechanics by means of integral transformations can be come to the system of functional equations in the complex plane decided by means of the Wiener–Hopf method [1]. The key problem of solution is the factorization of the system matrix coefficient. However now the only special matrix functions class of complex variable assuming exact factorization is known. The method of this matrices class factorization offered by G.N. Chebotarev and developed by A.A. Khrapkov [2] was successfully used in the solution of some problems on fracture mechanics, scattering theory of electromagnetic or elastic waves, contact problems. At the same time, the Chebotarev–Khrapkov method appeared insufficient in the case of more complicated matrices. This stimulated the development of matrix functions approximate factorization methods.

In this work the method of successive approximations is offered for the solution of the Wiener–Hopf functional equations system, using the presentation of the system matrix coefficient as the sum of two matrices, if the first matrix assumes the exact factorization, and the second one is assumed far less first matrix in the domain of system definition:

$$\mathbf{\Phi}^{+}(p)+\mathbf{F}(p)=\mathbf{G}(p)\mathbf{\Phi}^{-}(p),\ \ \mathbf{G}(p)=\mathbf{G}_0(p)+\mathbf{G}'(p)\ \ \left(\left|\mathbf{G}_0(p)\right| \gg \left|\mathbf{G}'(p)\right|\right)\ \ (p\in D), \qquad (1)$$

where $\mathbf{\Phi}^{+}(p), \mathbf{\Phi}^{-}(p)$ are unknown linear vectors, analytical in the domains D^{+} and D^{-} respectively, $D=D^{+}\cap D^{-}$; $\mathbf{F}(p)$ and $\mathbf{G}(p)$ are certain linear vector and square matrix, thus $\mathbf{G}_0(p)$ is factorized by means of analytical matrix in the domains D^{+} and D^{-}: $\mathbf{G}_0(p)=\mathbf{G}_0^{+}(p)\mathbf{G}_0^{-}(p)$. By means of the Wiener–Hopf method at the implementations of condition $p\to\infty$, $\left(\mathbf{G}_0^{+}(p)\right)^{-1}\mathbf{\Phi}_0^{+}(p)\to 0$ we obtain the solution of the system (1) in a zero approaching in the form of

$$\mathbf{\Phi}_0^{+}(p)=-\mathbf{G}_0^{+}(p)\mathbf{F}_0^{+}(p)\ \ (p\in D^{+}),\ \ \mathbf{\Phi}_0^{-}(p)=-\left(\mathbf{G}_0^{-}(p)\right)^{-1}\mathbf{F}_0^{-}(p)\ \ (p\in D^{-}),$$

$$\mathbf{F}_0^{\pm}(p)=\frac{1}{2\pi i}\int_{\gamma}\frac{\left(\mathbf{G}_0^{+}(z)\right)^{-1}\mathbf{F}(z)}{z-p}dz\ \ (\gamma\in D,\ \ p\in D^{\pm}),$$

where the degree "–1" denotes an inverse matrix, γ is an infinite line in the domain of system definition D.

In the first approximation, the equation (1) has the form

$$\mathbf{\Phi}_1^{+}(p)=\mathbf{G}_0(p)\mathbf{\Phi}_1^{-}(p)+\mathbf{G}'(p)\mathbf{\Phi}_0^{-}(p)\ \ (p\in D).$$

We introduce a vector function $\mathbf{F}_1(p)=-\mathbf{G}'(p)\mathbf{\Phi}_0^{-}(p)$ and find the first order correction by perturbation to the solution:

$$\mathbf{\Phi}_1^+(p) = -\mathbf{G}_0^+(p)\mathbf{F}_1^+(p) \ \ (p \in D^+), \ \ \mathbf{\Phi}_1^-(p) = -\left(\mathbf{G}_0^-(p)\right)^{-1}\mathbf{F}_1^-(p) \ \ (p \in D^-),$$

$$\mathbf{F}_1^{\pm}(p) = \frac{1}{2\pi i}\int_{\gamma} \frac{\left(\mathbf{G}_0^+(z)\right)^{-1}\mathbf{F}_1(z)}{z-p}dz = \frac{1}{2\pi i}\int_{\gamma} \frac{\left(\mathbf{G}_0^+(z)\right)^{-1}\mathbf{G}'(z)\left(\mathbf{G}_0^-(z)\right)^{-1}\mathbf{F}_0^-(z)}{z-p}dz \ \ (\gamma \in D, \ \ p \in D^{\pm}).$$

The n-th order corrections $\mathbf{\Phi}_n^{\pm}(p)$ are obtained in the same way as in the previous step. Summing up these corrections, we find the final solution of equation (1):

$$\mathbf{\Phi}^+(p) = -\mathbf{G}_0^+(p)\sum_{n=0}^{\infty}\mathbf{F}_n^+(p) \ (p \in D^+), \ \ \mathbf{\Phi}^-(p) = -\left(\mathbf{G}_0^-(p)\right)^{-1}\sum_{n=0}^{\infty}\mathbf{F}_n^-(p) \ \ (p \in D^-), \qquad (2)$$

$$\mathbf{F}_n^{\pm}(p) = \frac{1}{2\pi i}\int_{\gamma} \frac{\left(\mathbf{G}_0^+(z)\right)^{-1}\mathbf{G}'(z)\left(\mathbf{G}_0^-(z)\right)^{-1}\mathbf{F}_{n-1}^-(z)}{z-p}dz \ \ (\gamma \in \mathrm{D}, \ \ p \in D^{\pm}).$$

Formally the solution (2) can be considered an exact one. The condition of the correctness of the solution is the convergence of its series, that is, the implementation of inequality $\left|\mathbf{F}_n^{\pm}(p)\right| < \left|\mathbf{F}_{n-1}^{\pm}(p)\right|$. According to the definition $\mathbf{F}_n^{\pm}(p)$ in (2), this inequality is equivalent to the condition

$$\left|\left(\mathbf{G}_0^+(p)\right)^{-1}\mathbf{G}'(p)\left(\mathbf{G}_0^-(p)\right)^{-1}\right| < 1. \qquad (3)$$

At the same time, the practical application of formulas (2) for numerical calculations encounters an increase in the multiplicity of integrals in each next approximation. This circumstance forces us to be limited in (2) to a small number of terms, which are taken into account in calculations, and imposes a more strict restriction on the perturbation matrix instead of (3):

$$\left|\left(\mathbf{G}_0^+(p)\right)^{-1}\mathbf{G}'(p)\left(\mathbf{G}_0^-(p)\right)^{-1}\right| \ll 1.$$

As it can be seen from the previous consideration, the advantage of the suggested method is to avoid the need for factorization of the full coefficient $\mathbf{G}(p)$ of the initial equation (1). On the other hand, formally the solution (2) can be represented as the result of the action of some matrix operators:

$$\mathbf{\Phi}^{\pm}(p) = -\hat{\mathbf{G}}^{\pm}(p)\mathbf{F}_0(p) \ \ (p \in D^{\pm}),$$

which allows us to represent the factorization of the initial matrix coefficient $\mathbf{G}(p)$ as a result of the successive action of two analytic matrix operators:

$$\mathbf{G}(p) \to \hat{\mathbf{G}}^+(p)\hat{\mathbf{G}}^-(p).$$

In this interpretation, the considered method is close to some approximate methods of factorization of matrix functions, in particular, to the asymptotic method of factorization of the class of matrices proposed in [3], which at infinity tend to the unit matrix.

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